Research Design

09: statistical power, sample size, simulations

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Initial questions?

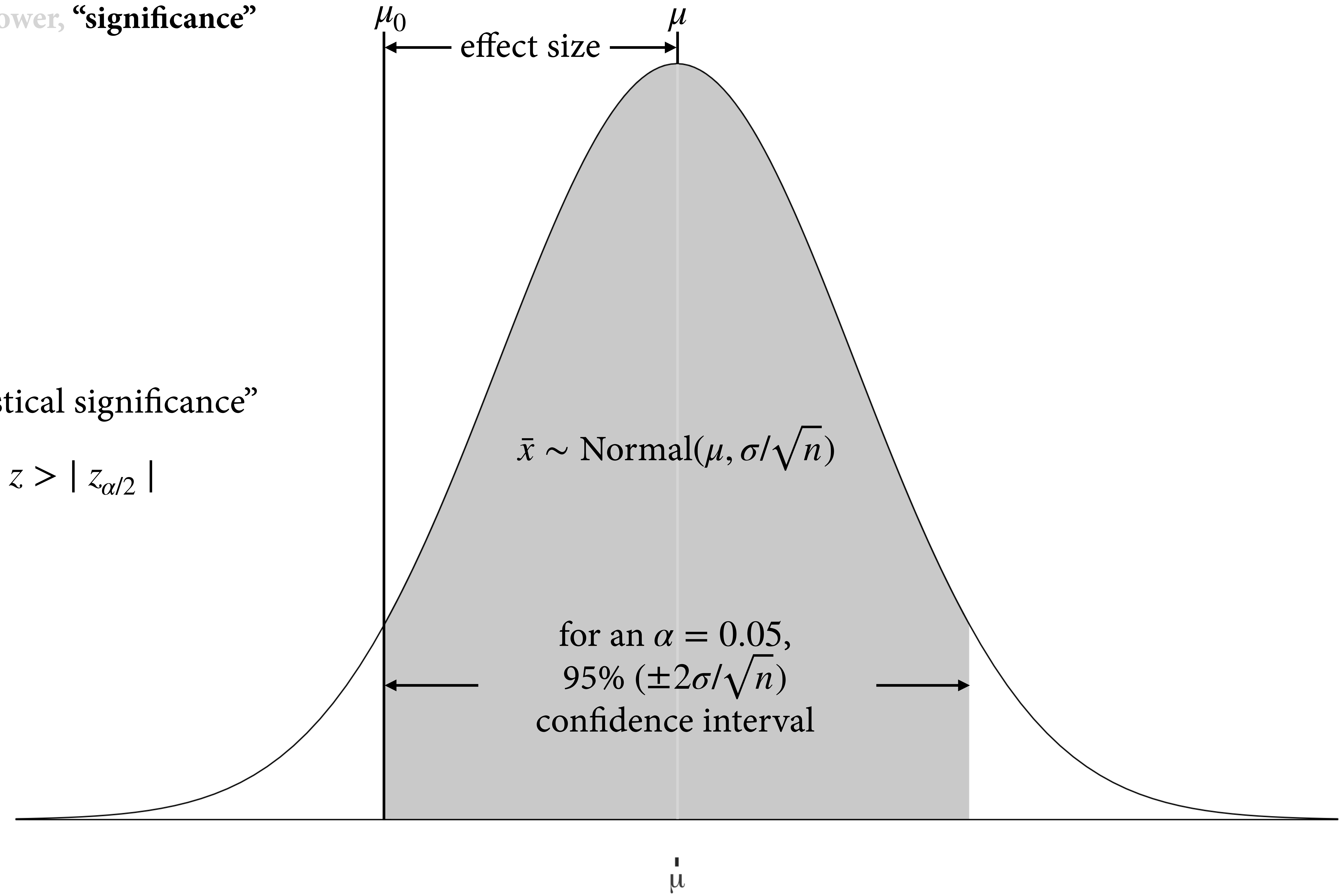
statistical power — probability of identifying statistical significance, given an effect

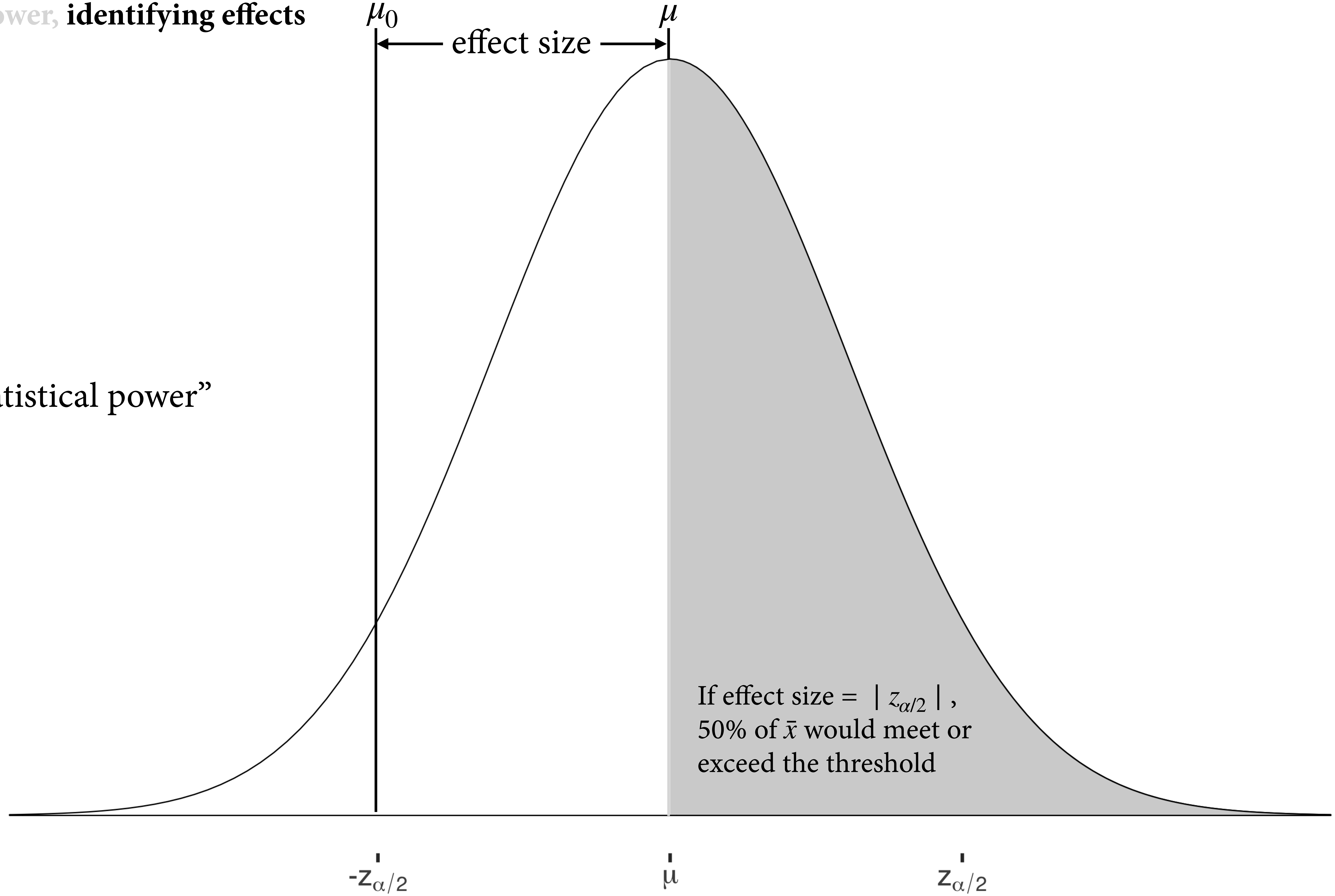
STATISTICAL POWER | the probability, before a study is performed, that a particular comparison will achieve "statistical significance" at some predetermined level (typically a p-value below 0.05), given some assumed true effect size.

NOTE | a typical threshold for statistical power $1 - \beta$ is 0.8 but — as with choosing a level of confidence α — choice β should inform good decisions.

“statistical significance”

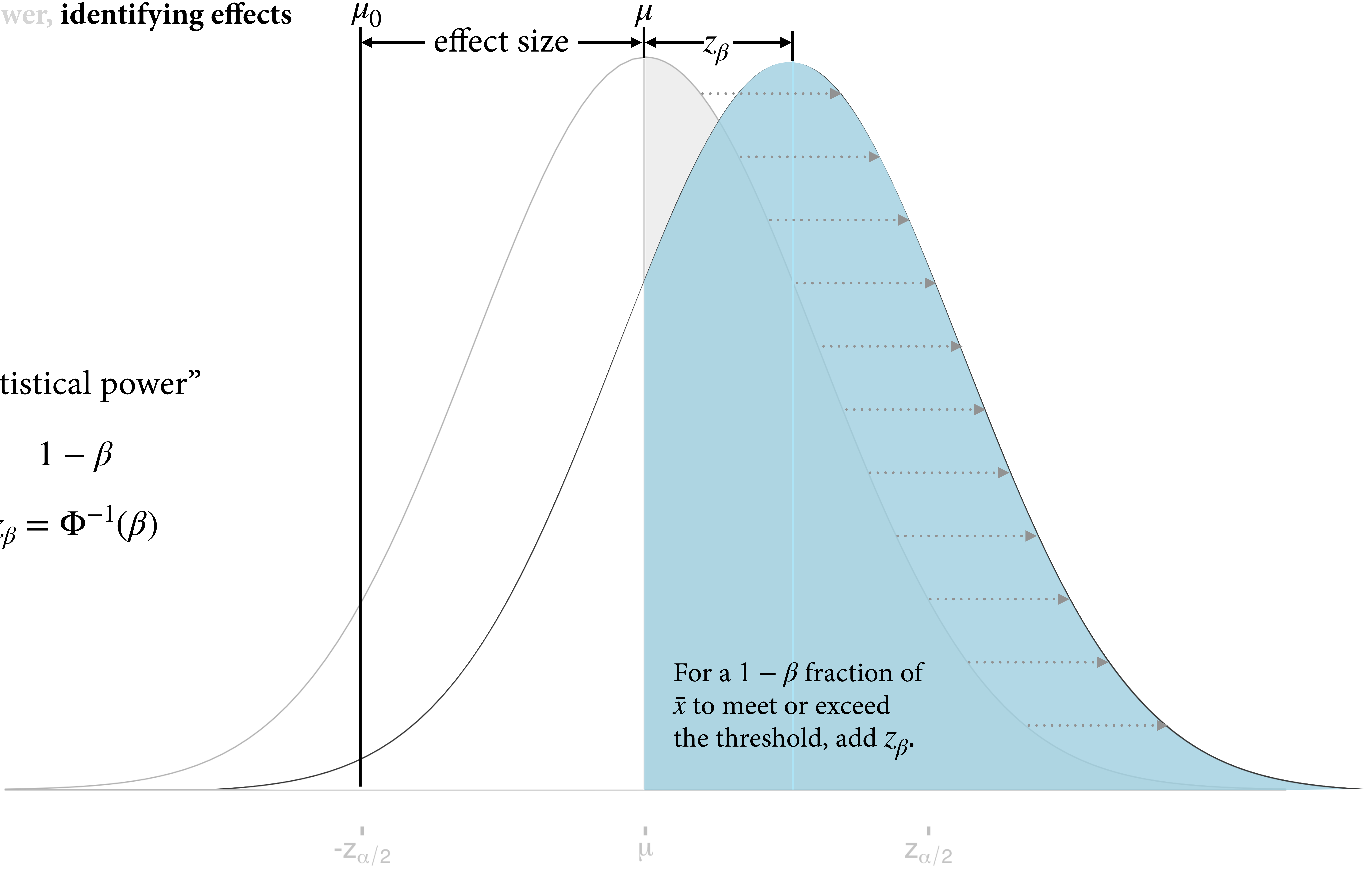
$z > | z_{\alpha/2} |$





"statistical power"

If effect size = $|\, z_{\alpha/2} \,|$,
50% of $\bar{x}$ would meet or
exceed the threshold



"statistical power"

$$1 - \beta$$

$$z_\beta = \Phi^{-1}(\beta)$$

 an example — calculating probability of finding an effect

Consider a hypothesis to test

$$H_0 : \mu = 0 \ , \ H_a : \mu > 0$$

Choose an appropriate test statistic and reference distribution (probability model)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > |\, z_\alpha \,| \ \text{ compared against } F_\Phi$$

Choose a meaningful effect size and variation to test

$$\bar{x} > \frac{\sigma}{\sqrt{n}} \,|\, z_\alpha \,| + \mu$$

Choose a sample size

$$n$$

Calculate the probability of identifying an underlying effect

$$p(\bar{x} > \frac{\sigma}{\sqrt{n}} \,|\, z_\alpha \,| + \mu) = F_\Phi(\frac{\sigma}{\sqrt{n}} \,|\, z_\alpha \,| + \mu)$$

sample size to achieve a specified probability of obtaining statistical significance

 estimating sample size to have p chance of finding an effect — solve for n

For a given $\mu - \mu_0$, α, and β: $\mu_0 + | Z_{\alpha/2} | \dfrac{s}{\sqrt{n}} = \mu - | Z_\beta | \dfrac{s}{\sqrt{n}}$

rearrange: $(| z_{\alpha/2} | + | z_\beta |) \cdot \dfrac{s}{\sqrt{n}} = \mu - \mu_0$

solve for n: $n = \left[\dfrac{(| z_{\alpha/2} | + | z_\beta |) \cdot s}{\mu - \mu_0} \right]^2$

estimating sample size — a toy example, estimate sample size for a proportion

For a given $\mu - \mu_0$, α, and β: $\mu_0 + |\, Z_{\alpha/2} \,| \dfrac{s}{\sqrt{n}} = \mu - |\, Z_\beta \,| \dfrac{s}{\sqrt{n}}$

Let $\mu = 0.6$, $\mu_0 = 0.5$, $\alpha = 0.05$, and $\beta = 0.2$.

rearrange: $(\,|\, z_{\alpha/2} \,| + |\, z_\beta \,|\,) \cdot \dfrac{s}{\sqrt{n}} = \mu - \mu_0$

$$(1.96 + 0.84)\frac{\sqrt{0.6(1-0.6)}}{\sqrt{n}} = 0.6 - 0.5$$

solve for n: $n = \left[\dfrac{(\,|\, z_{\alpha/2} \,| + |\, z_\beta \,|\,) \cdot s}{\mu - \mu_0} \right]^2$

$$n = \left[\frac{(1.96 + 0.84) \cdot 0.49}{0.1} \right]^2 = 196$$

simulations

Let $\mu = 0.6$, $\mu_0 = 0.5$, $\alpha = 0.05$, and try $\beta = \{0.2\ , 0.5\}$

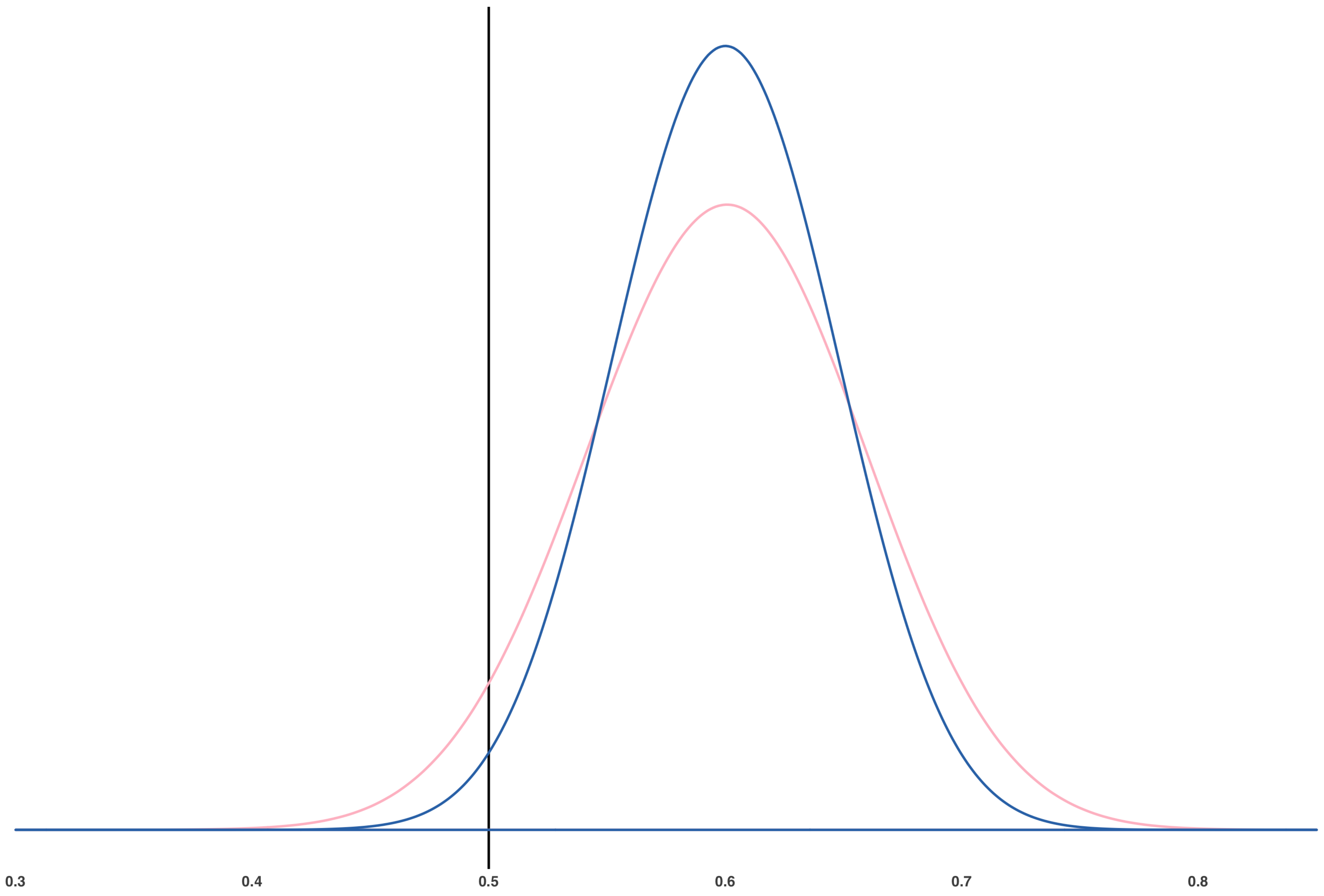
```
p0    <- 0.5
p     <- 0.6
alpha <- 0.05 / 2

z_alpha_2 <- qnorm(alpha, 0, 1, lower.tail = F)

# get n for 80 percent power
beta    <- 0.2
z_beta  <- qnorm(1 - beta, 0, 1, lower.tail = T)
n_pwr80 <- ( (z_alpha_2 + z_beta) * sqrt( p0 * (1 - p0) ) / (p - p0) ) ^ 2

# get n for 50 percent power
beta    <- 0.5
z_beta  <- qnorm(1 - beta, 0, 1, lower.tail = T)
n_pwr50 <- ( (z_alpha_2 + z_beta) * sqrt( p0 * (1 - p0) ) / (p - p0) ) ^ 2
```

Using $n_{\beta_{0.5}} = 96$ and $n_{\beta_{0.2}} = 196$, simulate experiments.

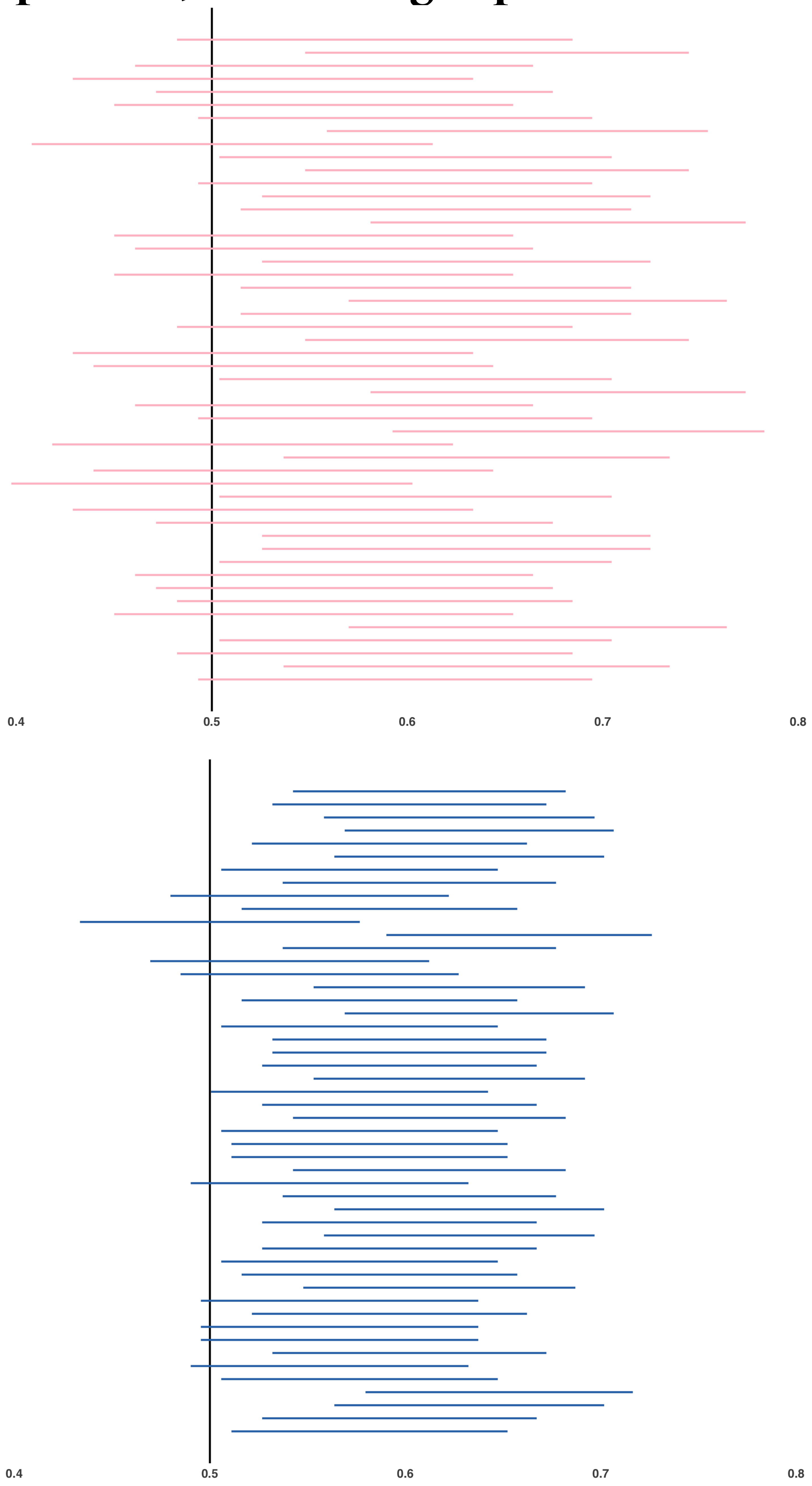
```
# simulate experiments
survey <- function(n, p) {
  x = ( rbinom(n = n, size = 1, prob = p) )
  x_bar = mean(x)
  se = sd(x) / sqrt(n)
  c(x_bar, se)
}


set.seed(1)
p_hat_96  <- replicate( 1e5, survey(n = n_pwr50, p) )

set.seed(1)
p_hat_196 <- replicate( 1e5, survey(n = n_pwr80, p) )
```

$\hat{p}$, with $n = 96$

$\hat{p}$, with $n = 196$



$$\hat{p} \pm 2 \cdot s/\sqrt{n}$$



References

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