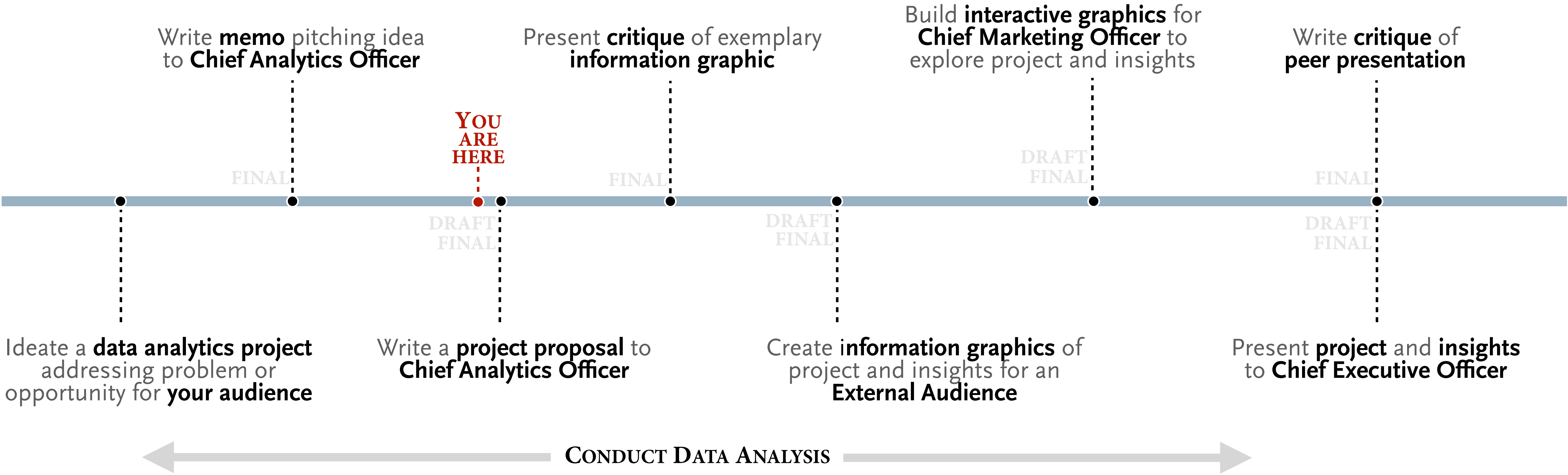


Storytelling with data

06 | the grammar of graphics; Doumont's three laws applied to data encoding; color; typologies



the grammar of graphics

the grammar of graphics, **grammar** — describes the form of relationships between [things]

GRAMMAR : 1.a. That department of the study of a [thing] which deals with its inflectional forms or other means of indicating the relations of [parts in things], and with the rules for employing these in accordance with established usage...

the grammar of graphics, statistical graphic specifications are expressed in six statements

DATA : a set of data operations that create variables from datasets

TRANSFORMATIONS : variable transformations (*e.g., rank, log, square root*)

SCALES : scale transformations (*e.g., linear, log, square root*)

COORDINATES : a coordinate system (*e.g., cartesian, polar*)

ELEMENTS : graphs (*e.g., points, lines, areas*) and their aesthetic attributes (*e.g., position, size, hue, saturation, luminance, opacity, orientation, shape*)

GUIDES : one or more guides (*axes, legends, etc.*)

the grammar of graphics, implementation example — ggplot2 (grammar of graphics)

```
# load grammar of graphics
library(ggplot2)
```

WILKINSON'S GRAMMAR

DATA TRANSFORMATIONS

ELEMENTS (WITH DATA) SCALES & GUIDES COORDINATES

```
# functions for data ink
```

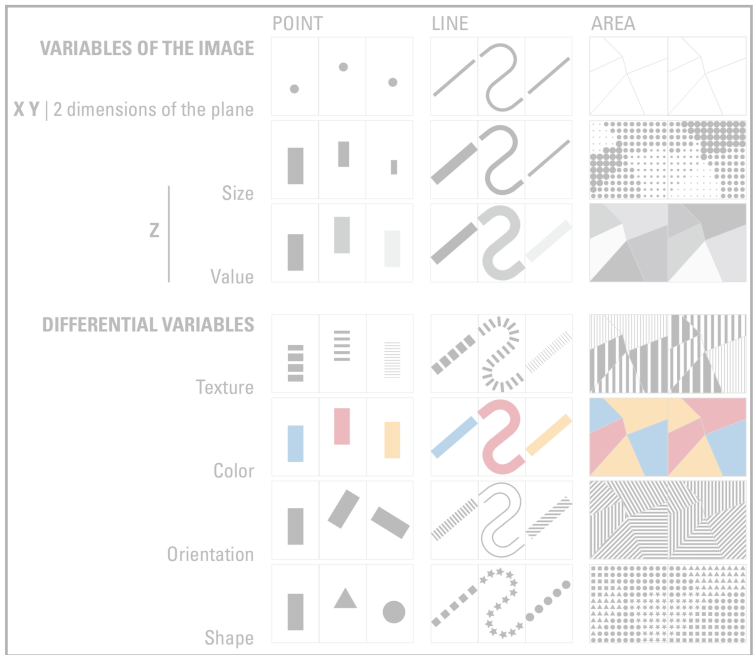
```
ggplot(data = <data>,
       mapping = aes(<aesthetic> = <variable>,
                     <...> = <...>) +
  geom_<type>(data = <...>, mapping = aes(<...>), <...>) +
  scale_<mapping>_<type>(<...>) +
  coord_<type>(<...>) +
  facet_<type>(<...>) +
  <...> +
```

```
# functions for non-data ink
```

GUIDES

```
labs(<...>) +
theme(<...> = <...>) +
annotate(<...>) +
<...>
```

the grammar of graphics, implementation example — ggplot2 (grammar of graphics)



WILKINSON’S GRAMMAR

DATA
TRANSFORMATIONS

ELEMENTS (WITH DATA)
SCALES & GUIDES
COORDINATES

GUIDES

```
# load grammar of graphics
library(ggplot2)
```

```
# functions for data ink
```

```
ggplot(data = <data>,
       mapping = aes(<aesthetic> = <variable>,
                     <...> = <...>)) +
  geom_<type>(data = <...>, mapping = aes(<...>), <...>) +
  scale_<mapping>_<type>(<...>) +
  coord_<type>(<...>) +
  facet_<type>(<...>) +
  <...> +
```

BERTIN’S VARIABLES +
x, y, z, size, color, fill, alpha,
angle, shape, group ... and more,
depends on the geometry used.

BERTIN’S ELEMENTS +
point, line, segment,
circle, rect, polygon,
path ... and many more!

```
# functions for non-data ink
```

```
labs(<...>) +
theme(<...> = <...>) +
annotate(<...>) +
<...>
```

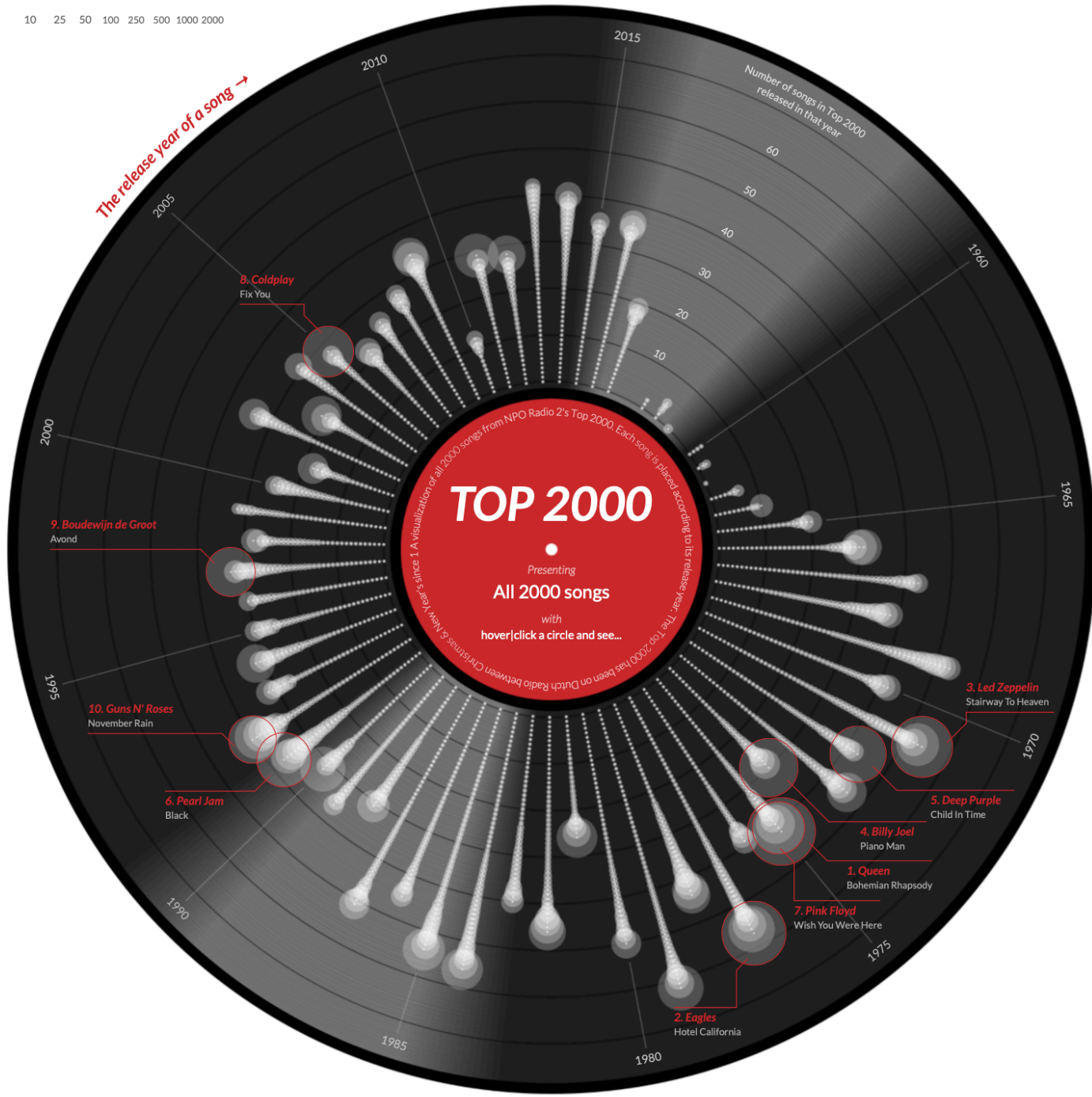
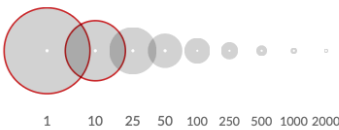
element_blank()
element_line(<...> = <...>)
element_rect(<...> = <...>)
element_text(<...> = <...>)

the grammar of graphics, elements in *layers* — example

Let The Music Play

All songs from the TOP 2000 of 2017 according to their release years

Position in the Top 2000



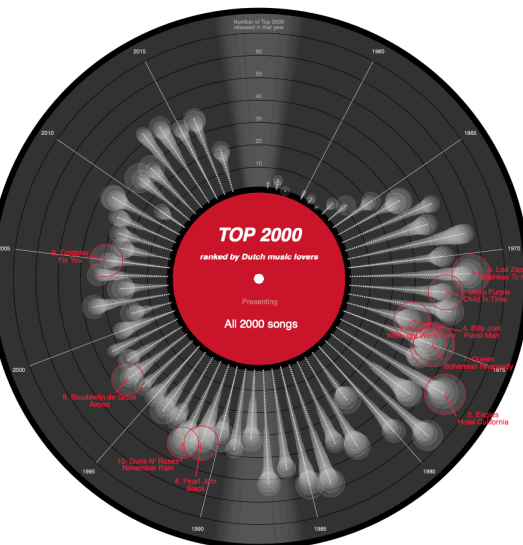
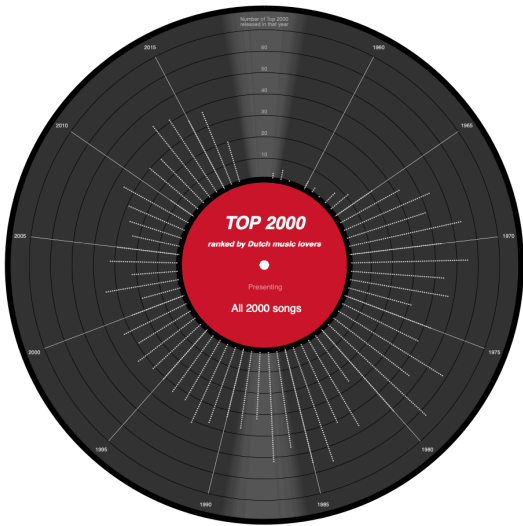
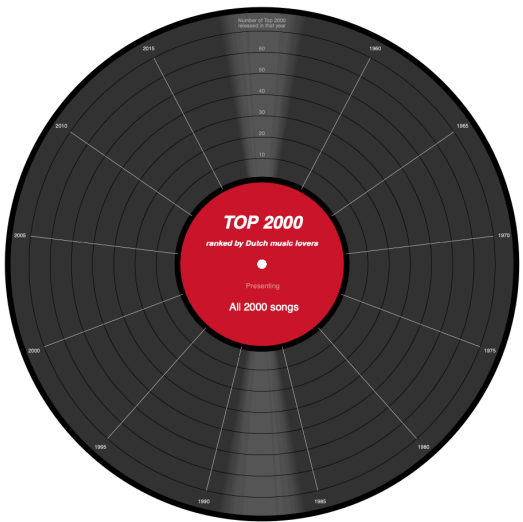
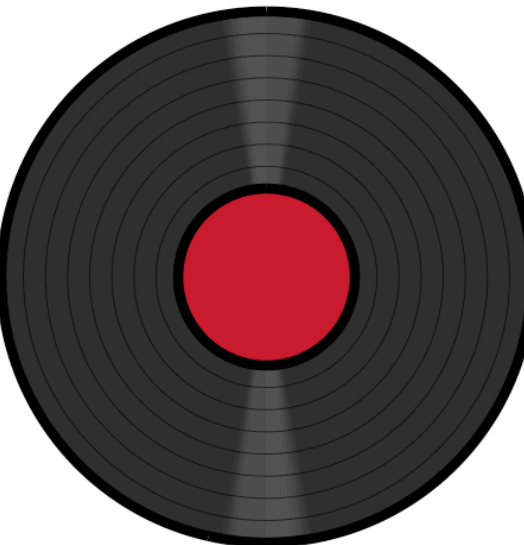
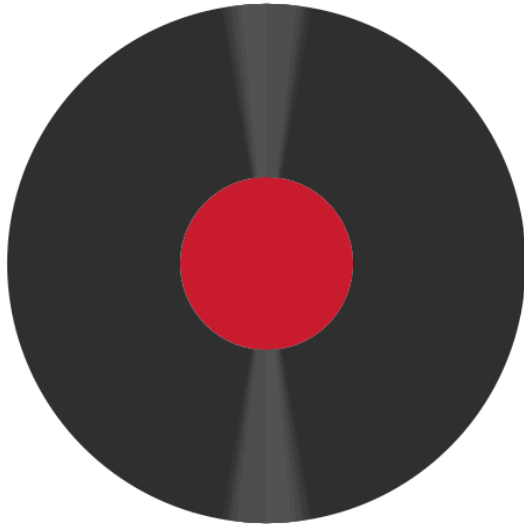
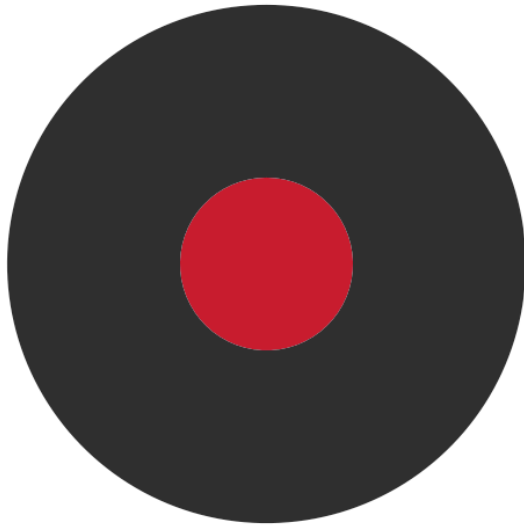
When hovering (or clicking on mobile) a circle all the songs of that artist are highlighted and connected by a line. When the song happens to be a collaboration between two artists, you'll see multiple lines, one for each artist.

Created by Nadieh Bremer | Visual Cinnamon

Data from NPO Radio 2

Ga naar de Nederlandse versie

Check out my Top 2000 visuals from 2015 & 2016



Doumont's *three laws of communication* applied to data encoding

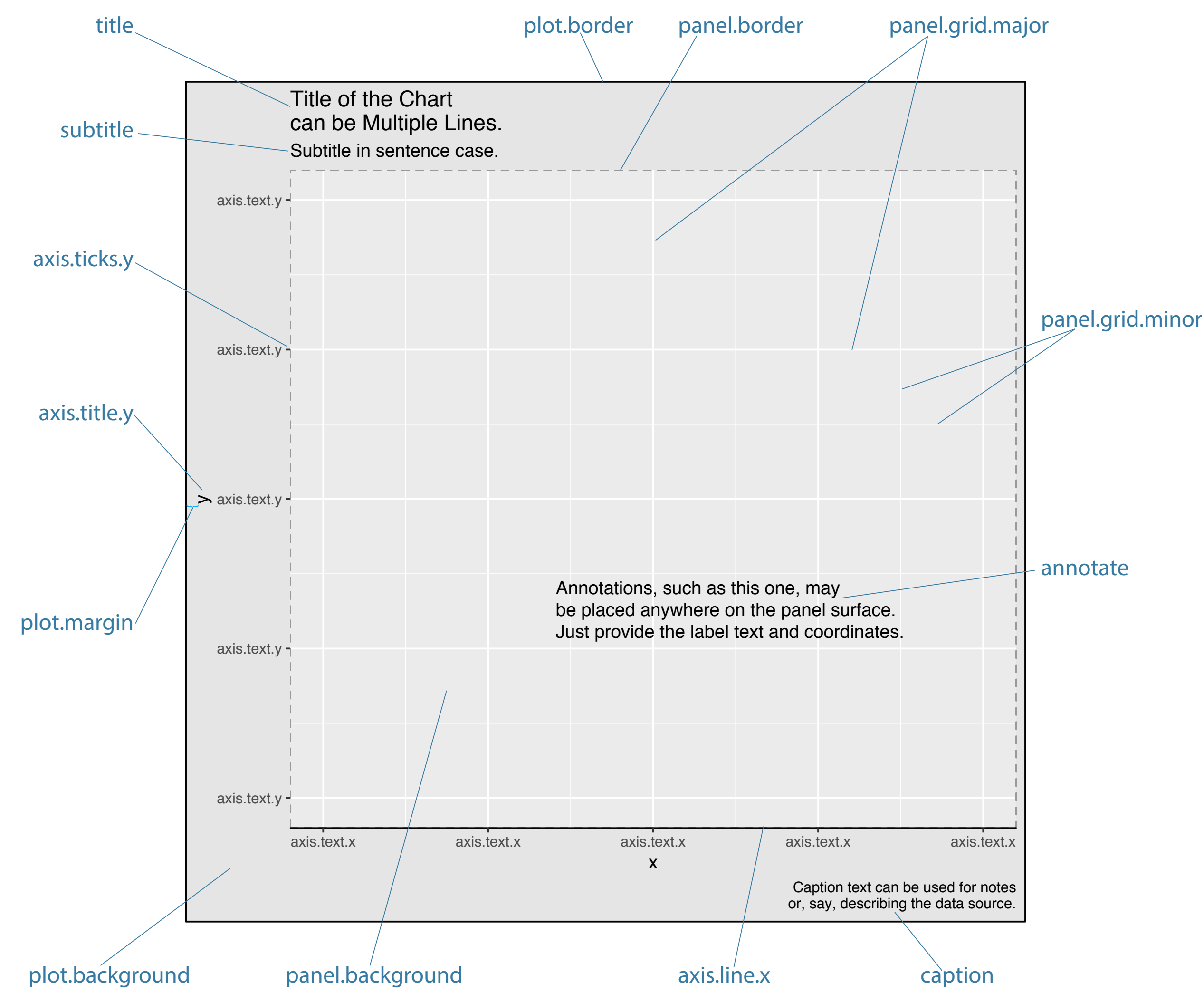
Doumont applied to data encoding, Doumont's three laws of communication

Adapt to your audience

Maximize the signal-to-noise ratio

Use effective redundancy

Doumont applied to data encoding, non-“data ink”



Doumont applied to data encoding, non-“data ink” — example functions to draw non-data ink in ggplot2

```
# load grammar of graphics
library(ggplot2)
```

```
p <-
```

```
# functions for data ink
```

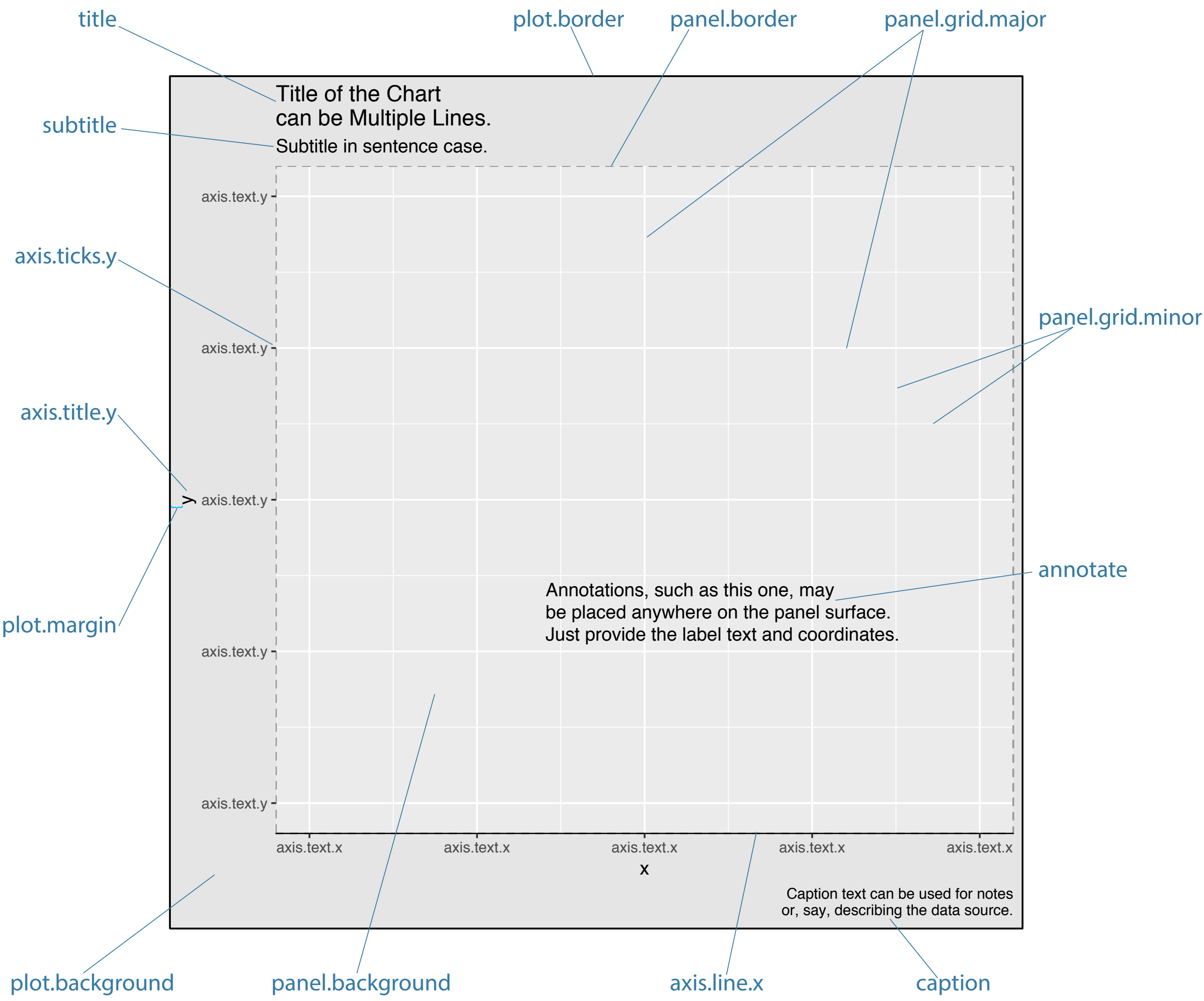
```
ggplot(data = <data>,
       mapping = aes(<aesthetic> = <variable>,
                     <aesthetic> = <variable>,
                     <...> = <...>)) +

geom_<type>(<...>) +
scale_<mapping>_<type>(<...>) +
coord_<type>(<...>) +
facet_<type>(<...>) +
<...> +
```

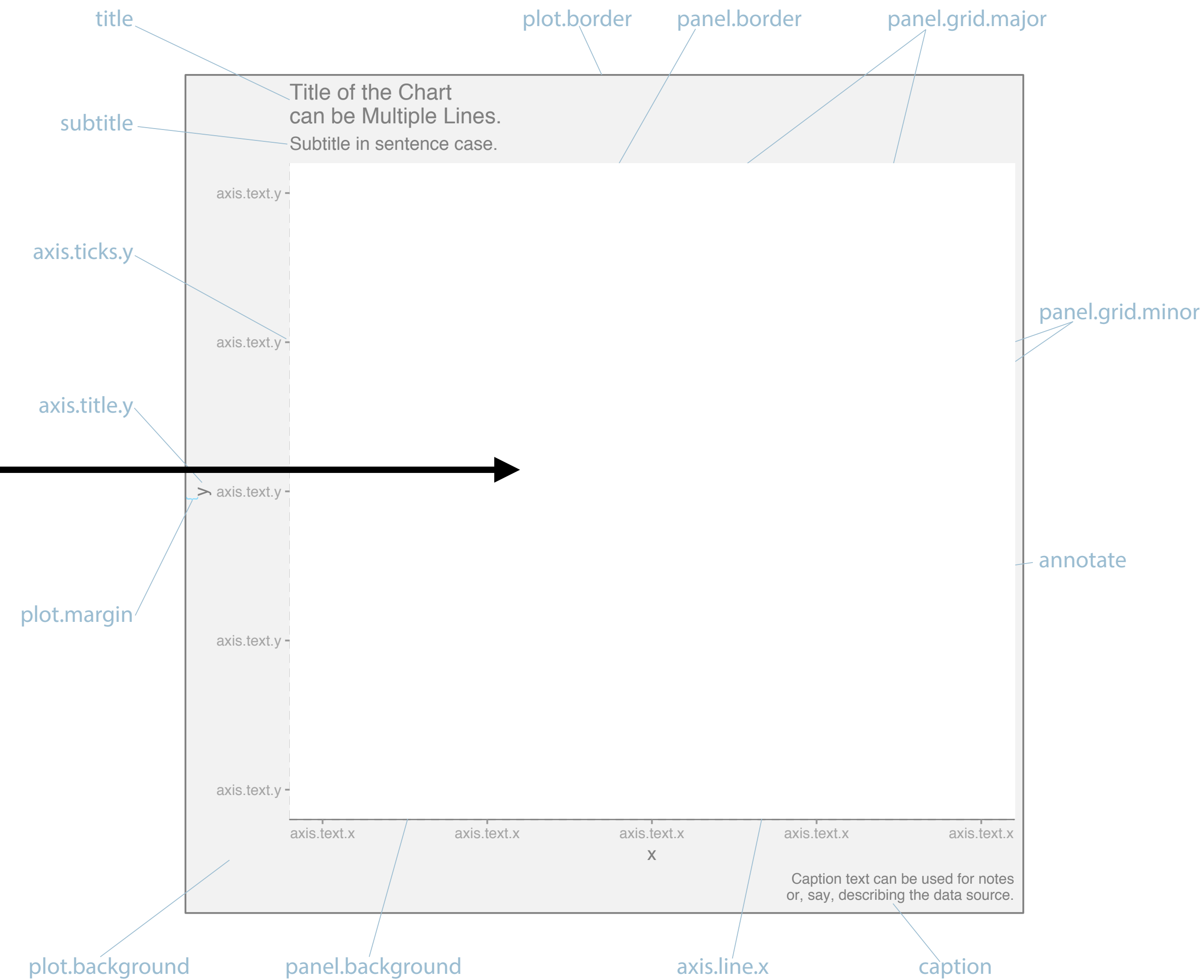
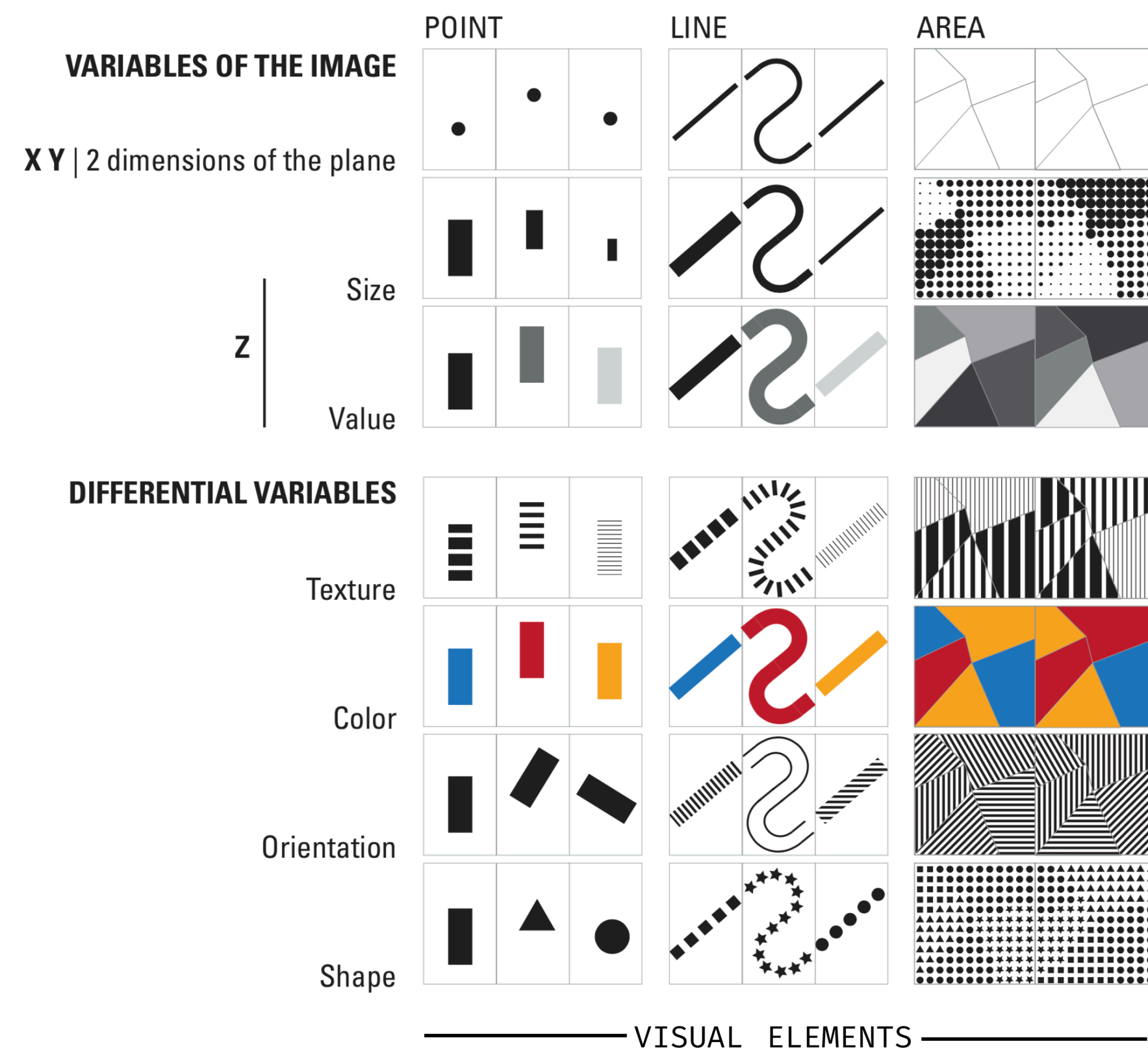
functions for non-data ink

labs(<...>) +
theme(<...> = <...>) +
annotate(<...>) +
<...>

element_blank()
element_line(<...> = <...>)
element_rect(<...> = <...>)
element_text(<...> = <...>)



Doumont applied to data encoding, “data ink” — Jacques Bertin’s visual channels for encoding data



Doumont applied to data encoding, the data ink — example functions to draw encoded data in ggplot2

```
# load grammar of graphics
library(ggplot2)
```

```
p <-
```

```
# functions for data ink

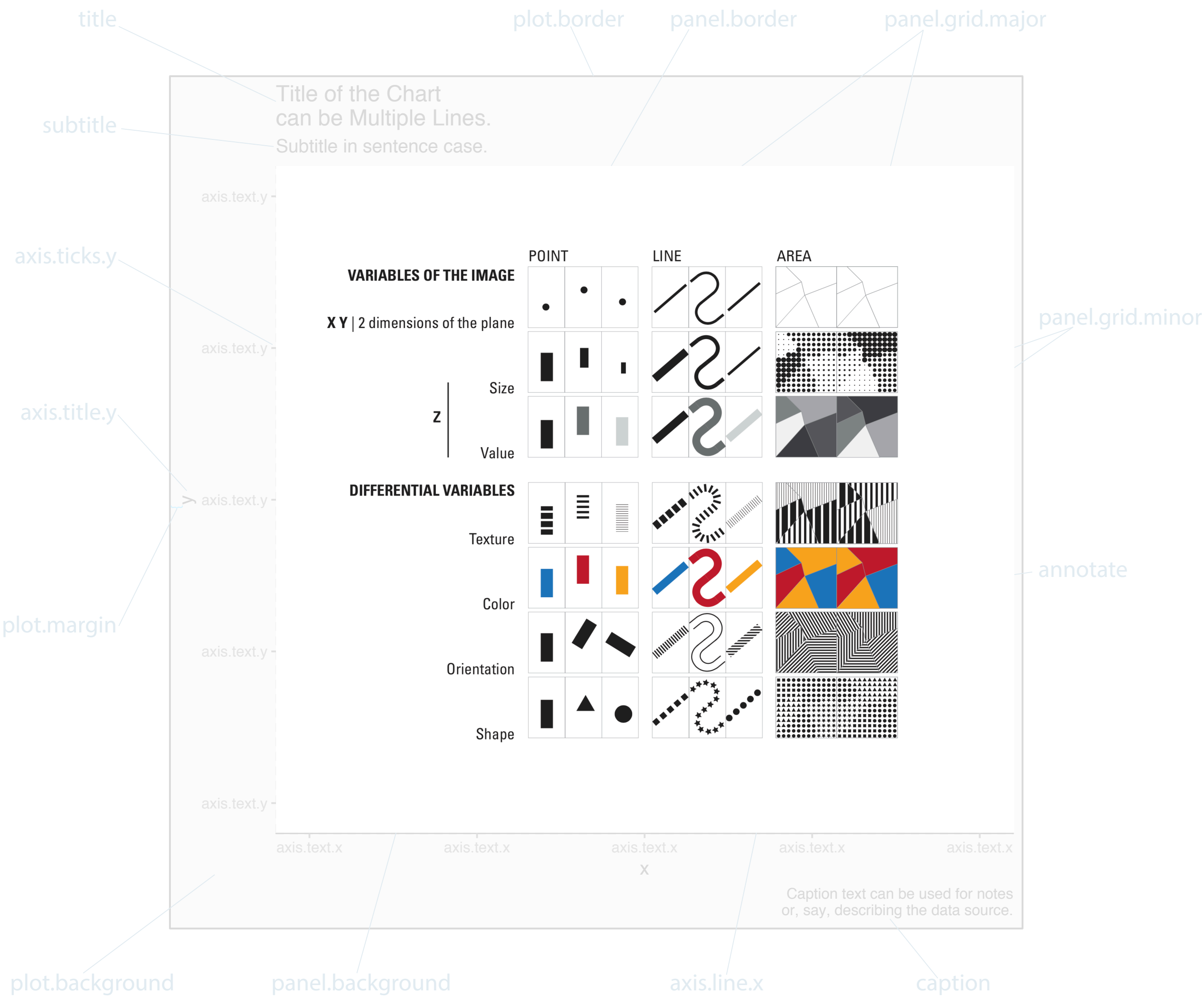
ggplot(data = <data>,
       mapping = aes(<aesthetic> = <variable>,
                     <aesthetic> = <variable>,
                     <....> = <....>) +

geom_<type>(<....>) +
scale_<mapping>_<type>(<....>) +
coord_<type>(<....>) +
facet_<type>(<....>) +
<....> +
```

```
# functions for non-data ink
```

```
labs(<....>) +
theme(<....> = <....>) +
annotate(<....>) +
<....>
```

```
element_blank()
element_line(<....> = <....>)
element_rect(<....> = <....>)
element_text(<....> = <....>)
```



Doumont applied to data encoding, Tufte — data-ink maximization, *within reason*

data-ink ratio

=

data-ink

total ink used to print the graphic

=

proportion of a graphic’s ink devoted to the non-redundant display of data-information

=

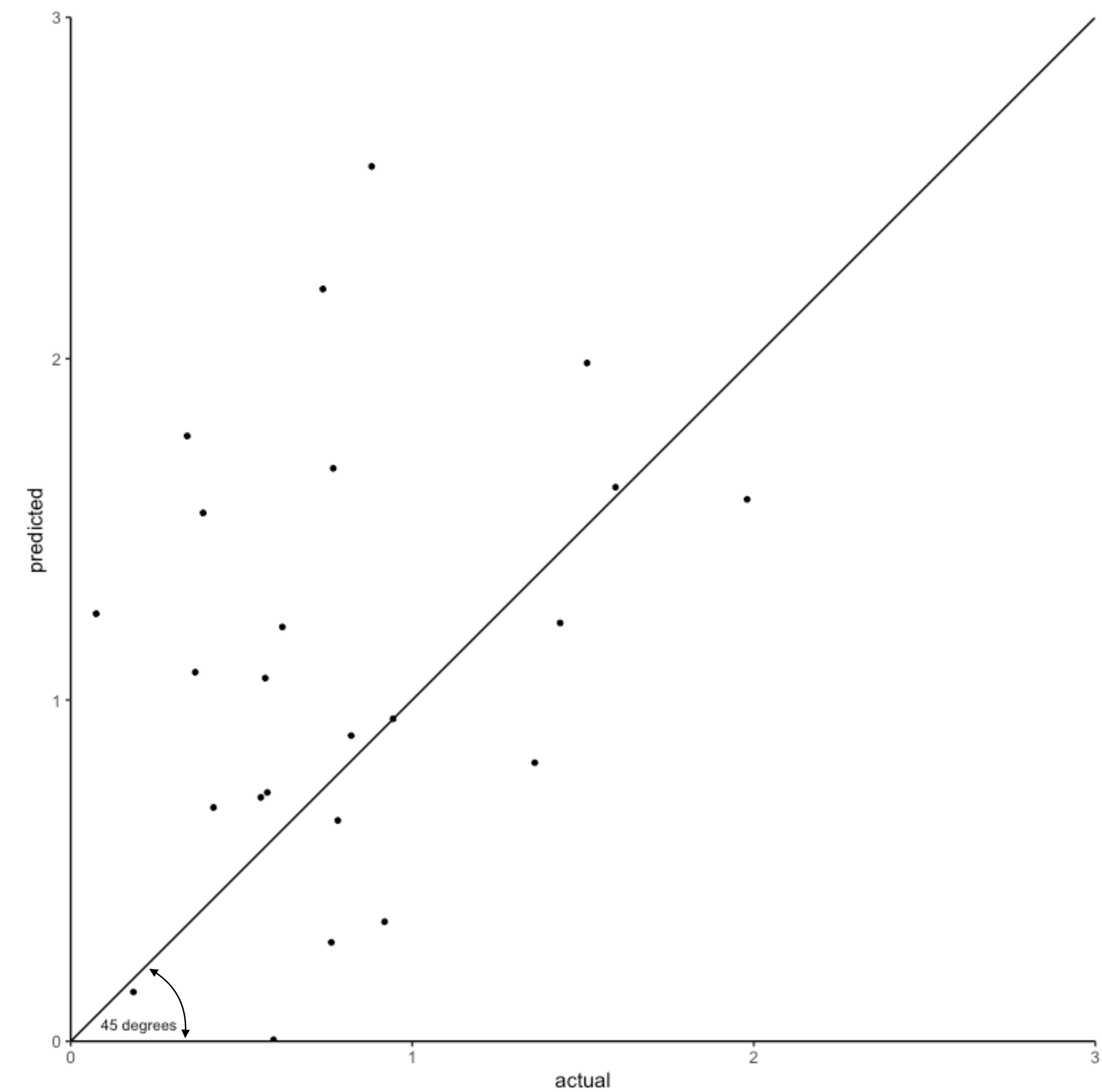
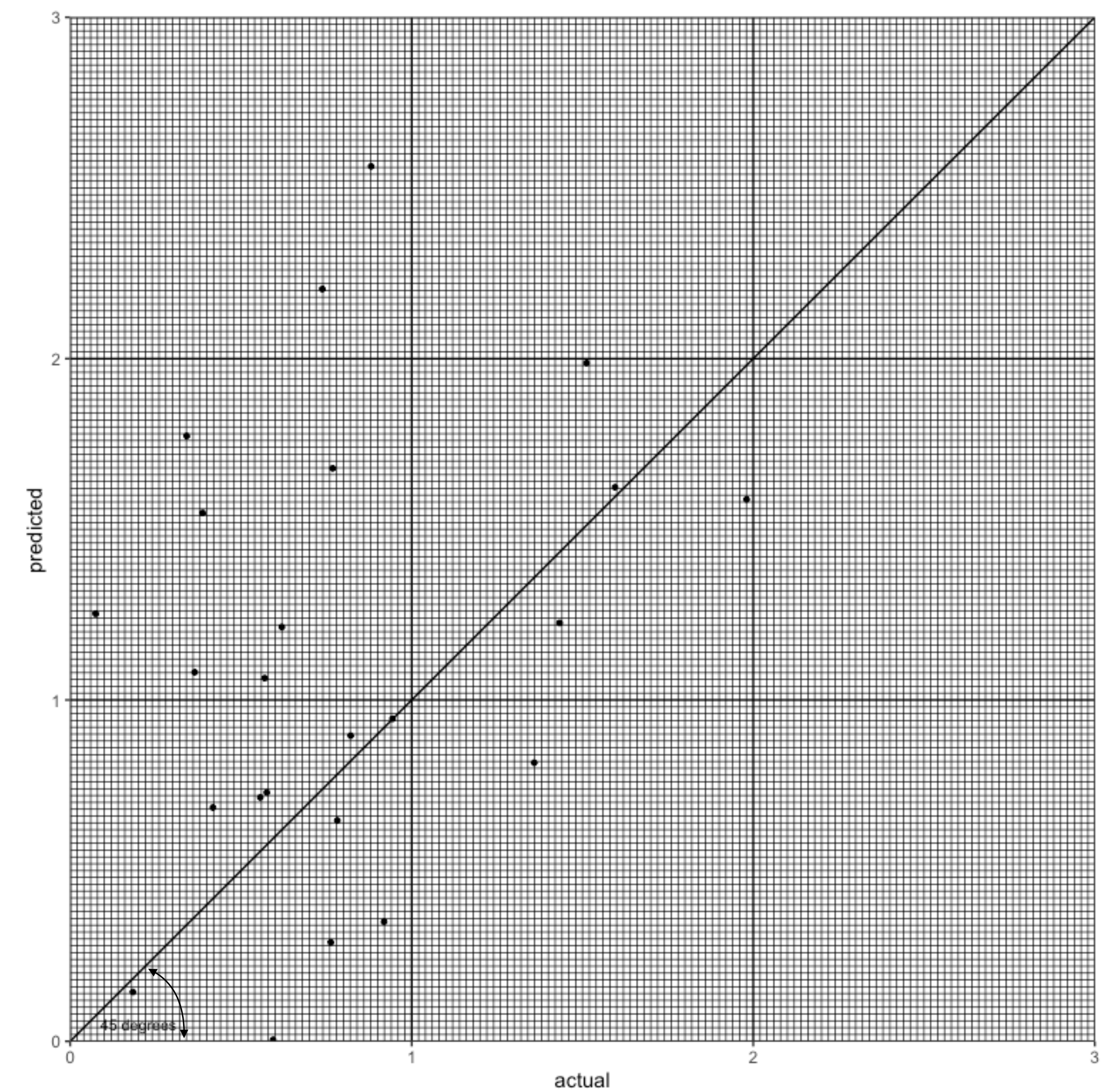
1.0 – proportion of a graphic that can be erased without loss of data-information

“The principle *helps conduct experiments* in graphical design.”

— Tufte, Edward, *The Visual Display of Quantitative Information*

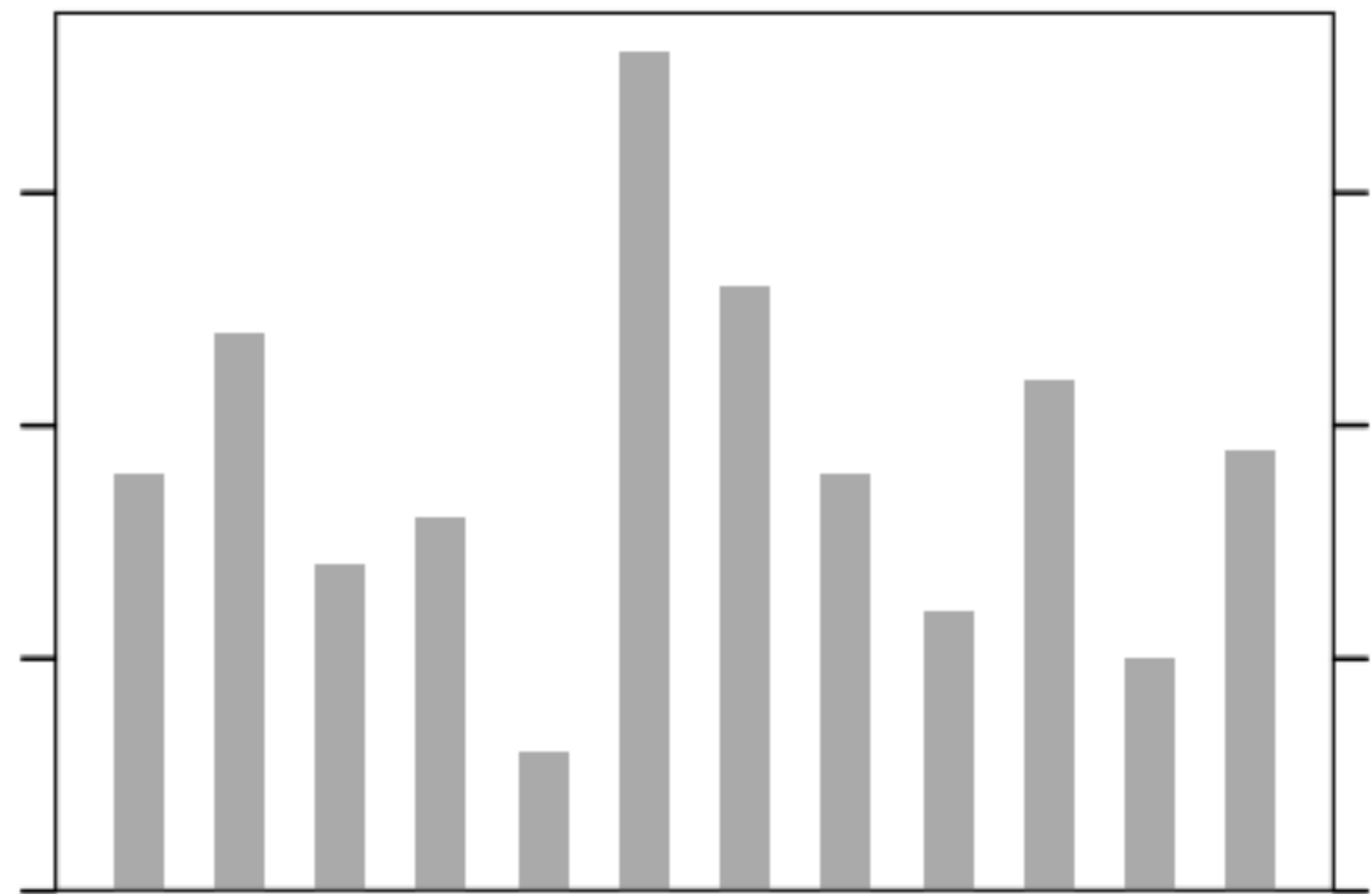
Doumont — “maximize the signal-to-noise ratio”

Doumont applied to data encoding, example — removing gridlines increases the data-ink ratio

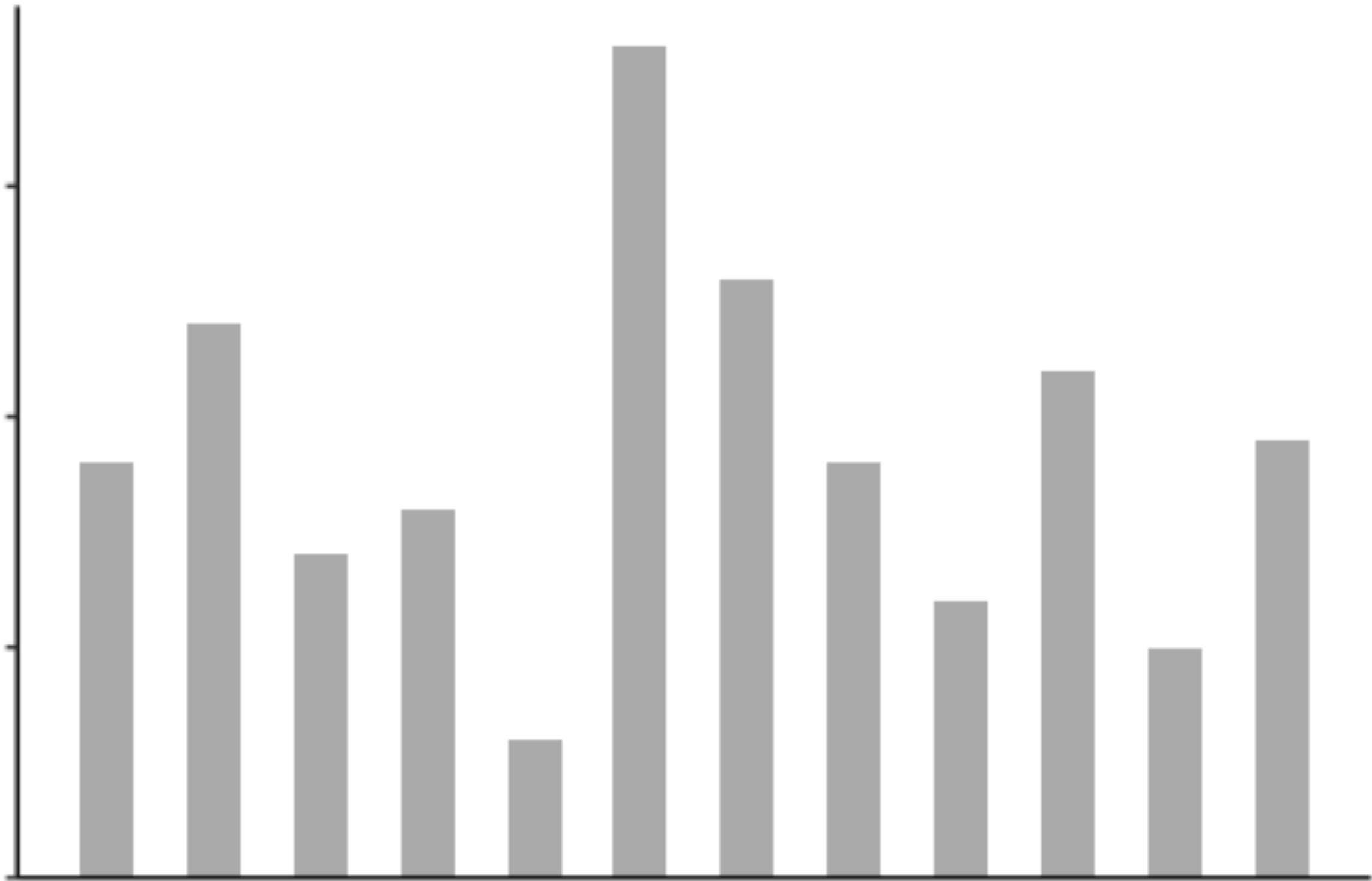


Doumont — “maximize the signal-to-noise ratio”

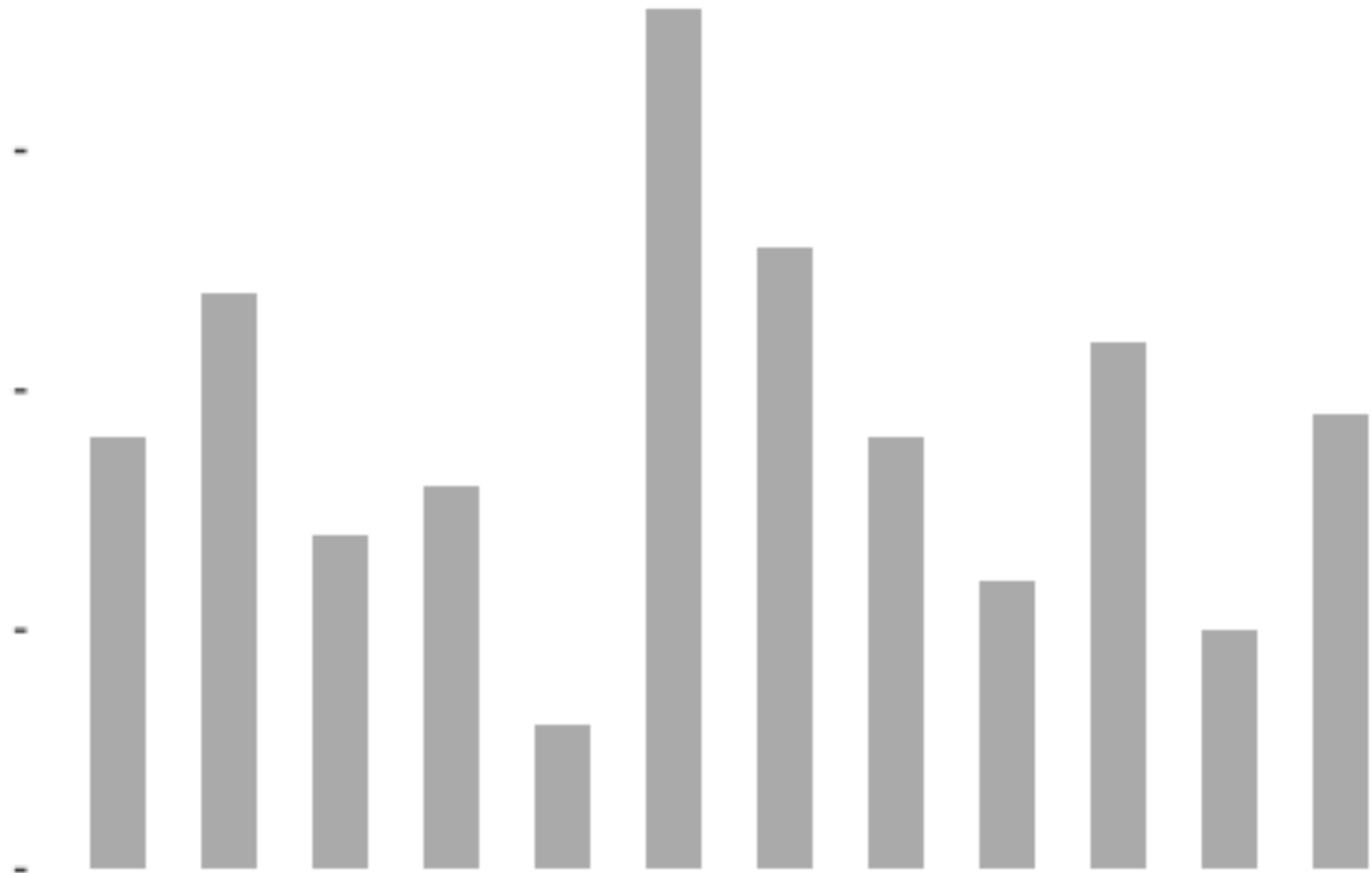
Doumont applied to data encoding, **experimentation** — redesigning a bar chart



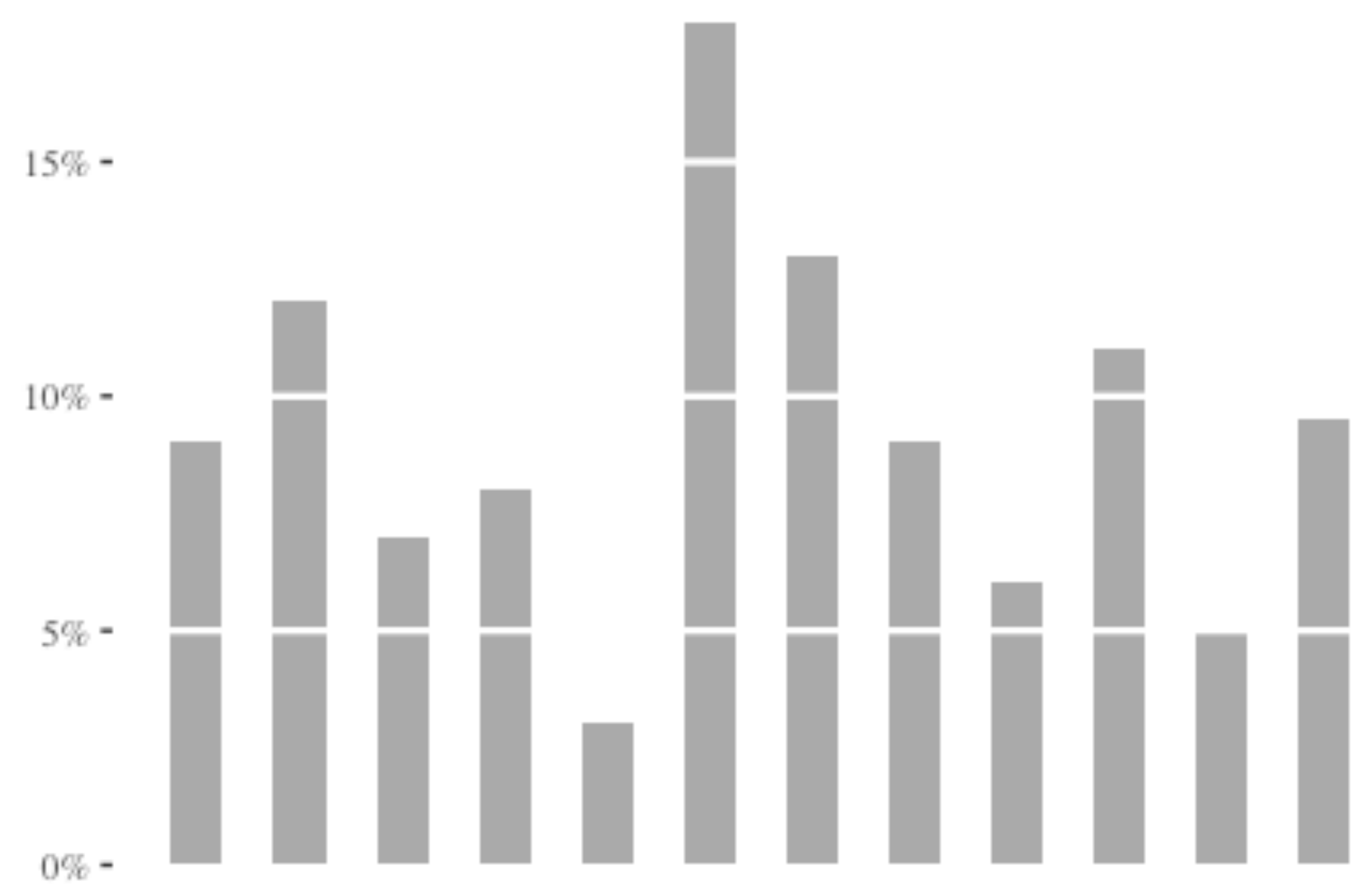
Doumont applied to data encoding, **experimentation** — redesigning a bar chart



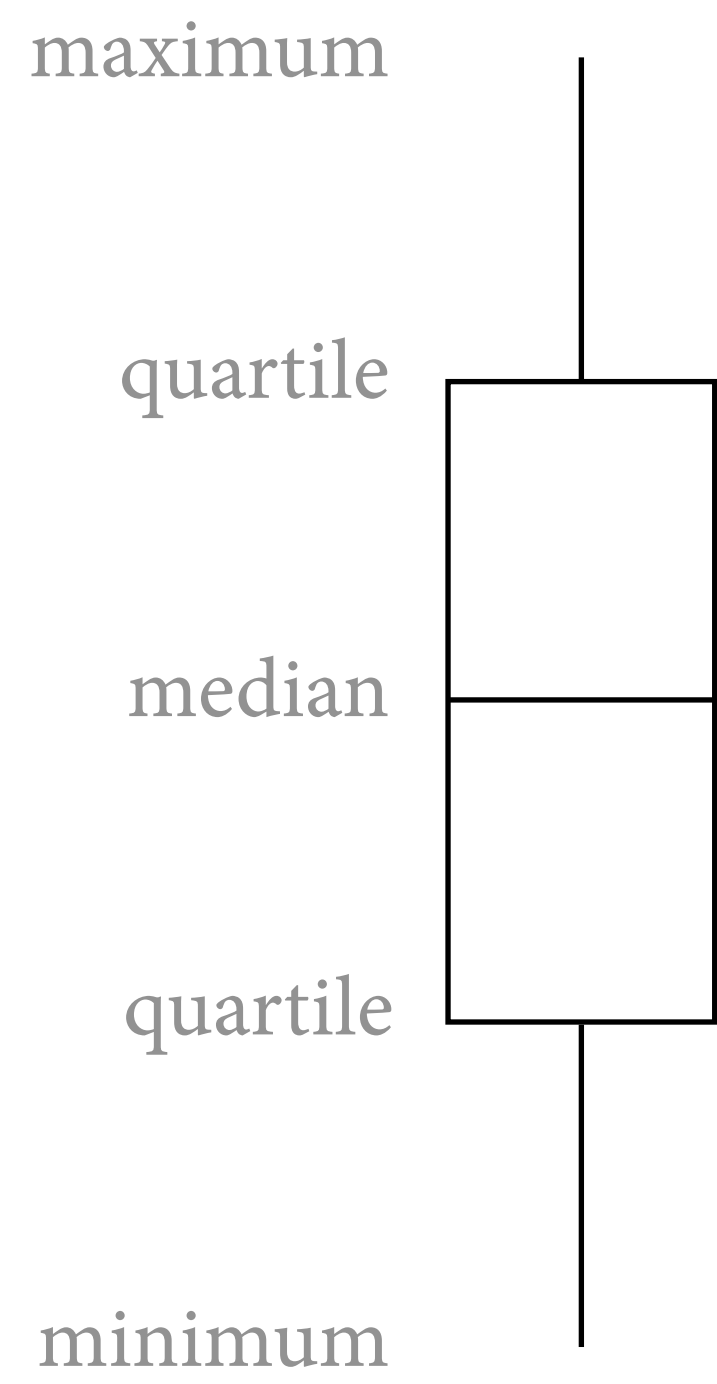
Doumont applied to data encoding, experimentation — redesigning a bar chart



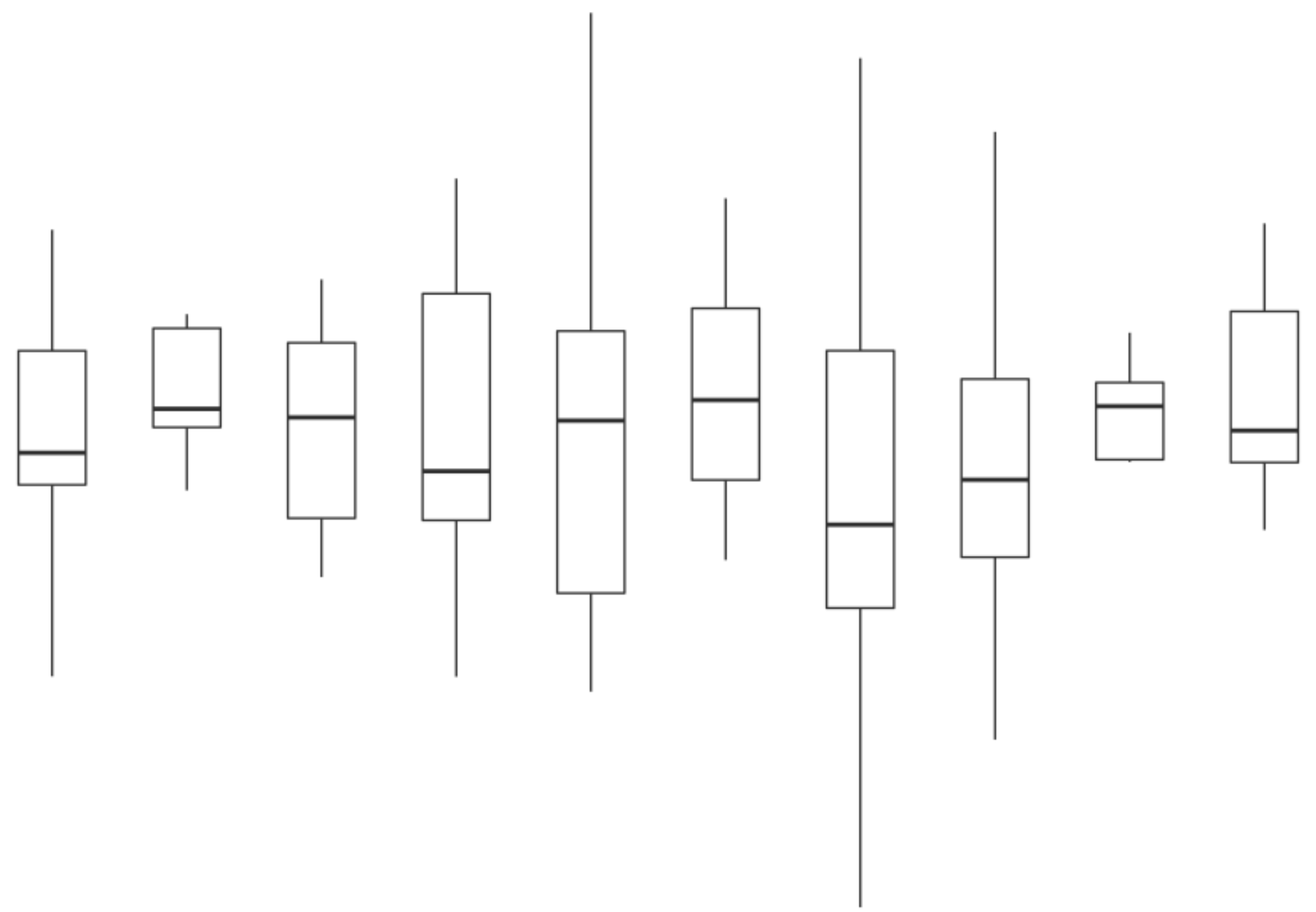
Doumont applied to data encoding, experimentation — redesigning a bar chart



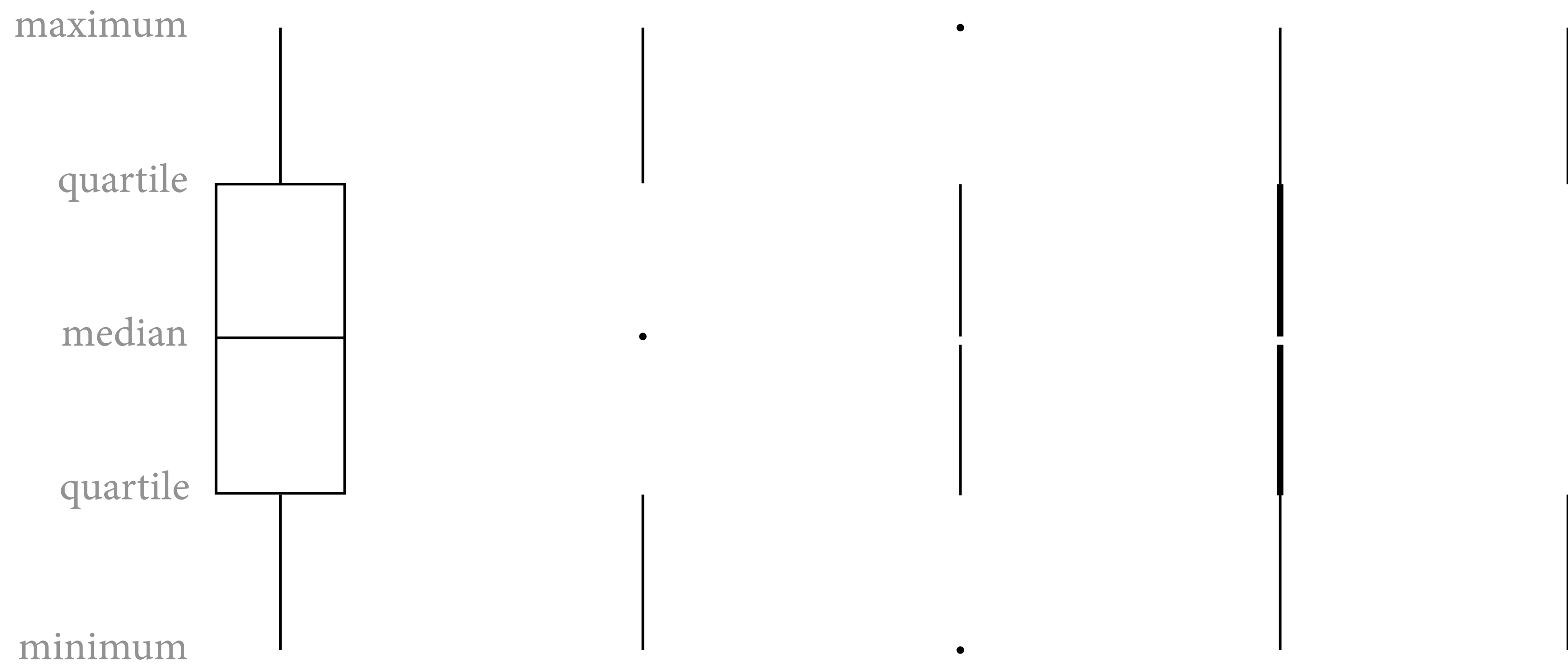
Doumont applied to data encoding, experimentation — redesigning John Tukey’s box plot



Doumont applied to data encoding, **experimentation** — redesigning John Tukey’s box plot



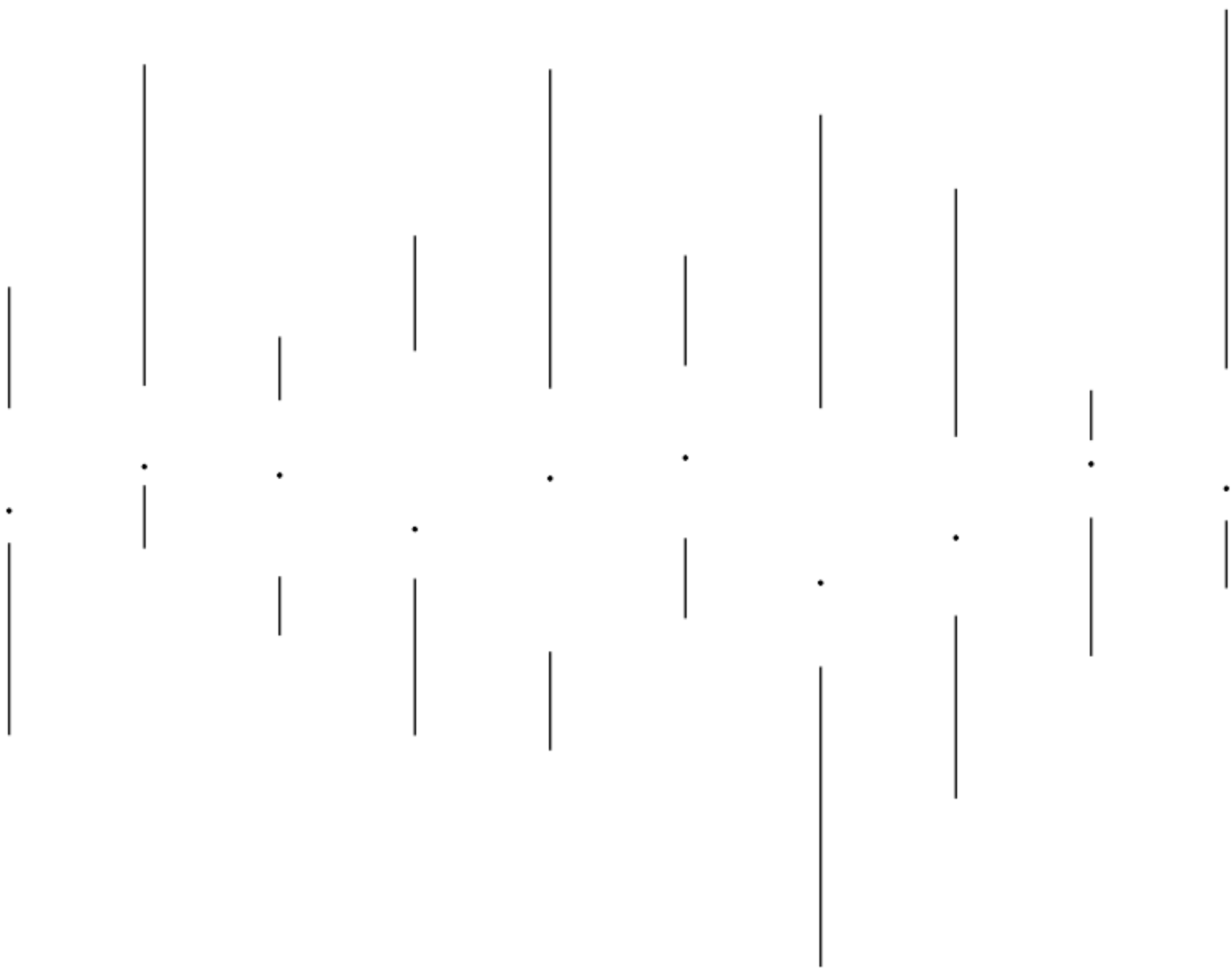
Doumont applied to data encoding, experimentation — redesigning John Tukey’s box plot



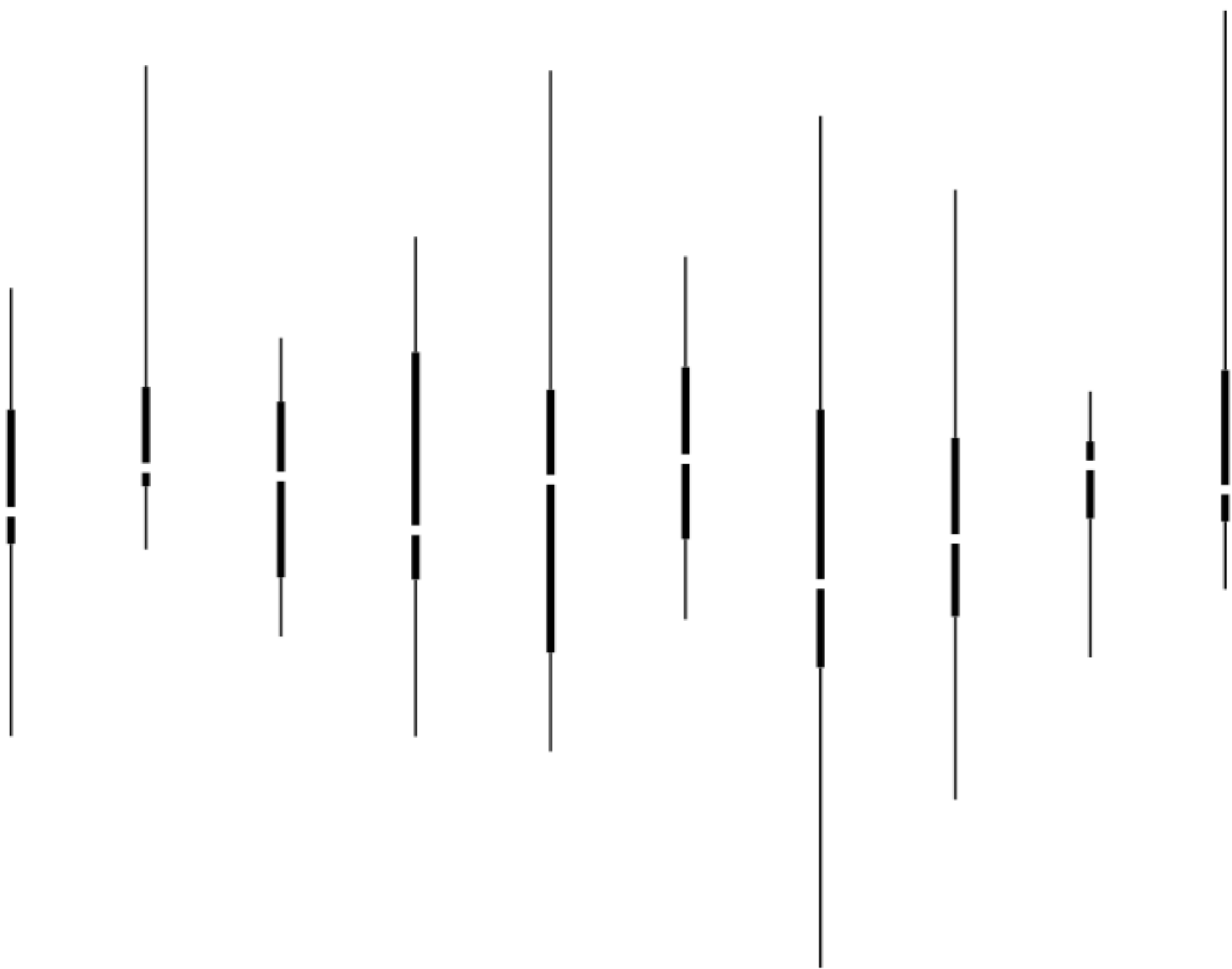
“In these revisions of the box plot, . . . the best overall arrangement naturally also rests on statistical and aesthetic criteria — in other words, the procedure is one of *reasonable* data-ink maximizing.”

— Tufte, Edward, *The Visual Display of Quantitative Information*

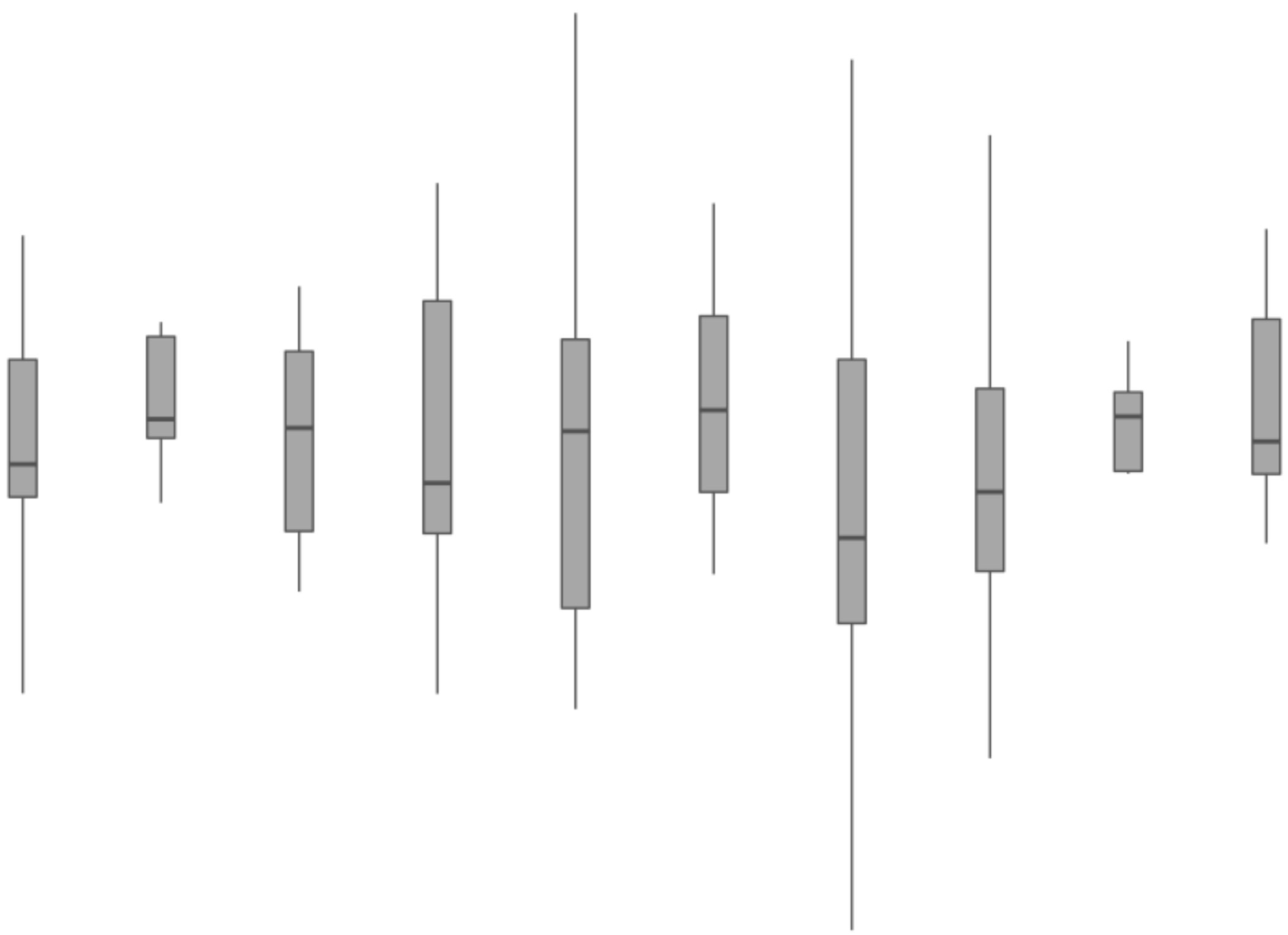
Doumont applied to data encoding, **experimentation** — redesigning John Tukey’s box plot



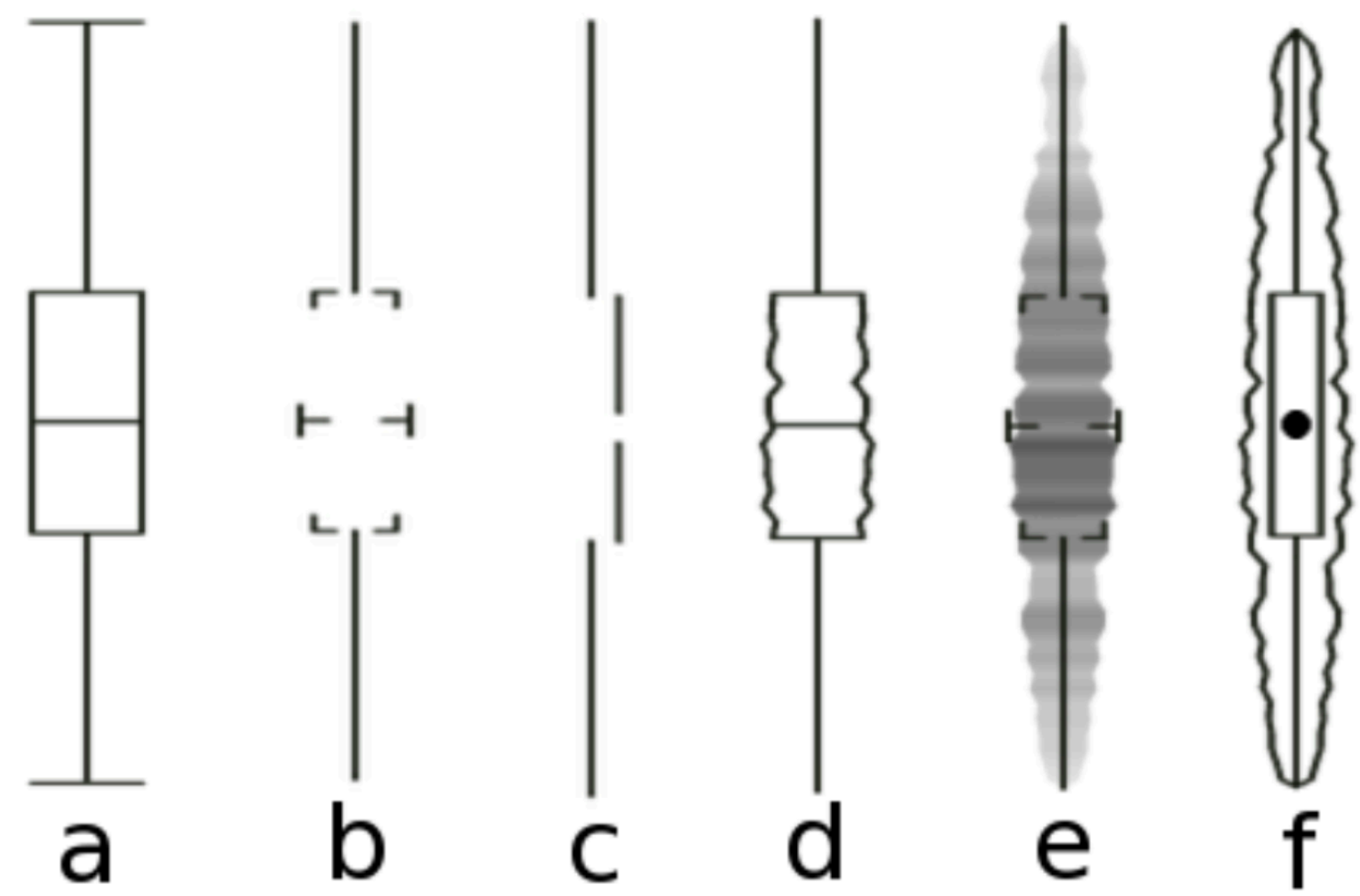
Doumont applied to data encoding, **experimentation** — redesigning John Tukey’s box plot



Doumont applied to data encoding, experimentation — redesigning John Tukey’s box plot



Doumont applied to data encoding, which works best? — *iterative process of creating, questioning, testing!*



One empirical experiment, asking participants to state data characteristics, given one of these randomized versions of the boxplots shown here, suggested that one of Tufte’s erasures (option C) was most cognitively difficult for viewers to interpret.

Cues like labels and gridlines, together with some strictly superfluous embellishment of data points or other design elements, may sometimes be an aid rather than an impediment to interpretation.

Adapt to your audience.

Doumont — “adapt to your audience”

Doumont applied to data encoding, **data-ink maximization** — **one of many design considerations**

“Maximizing data ink (within reason) **is but a single dimension of a complex and multivariate design task.**

The principle helps conduct
experiments in graphical design.

Some of those
experiments will succeed.

There remain, however, many **other considerations** in the design of statistical graphics — not only of efficiency, but also of **complexity, structure, density, and even beauty.**”

— Tufte, Edward, *The Visual Display of Quantitative Information*

Doumont applied to data encoding, which works best? — *iterative process of creating, questioning, testing!*

**Prototypes should
emphasize speed over polish.**

Design is a search problem

**Get fresh eyes frequently.
Invite criticism.**

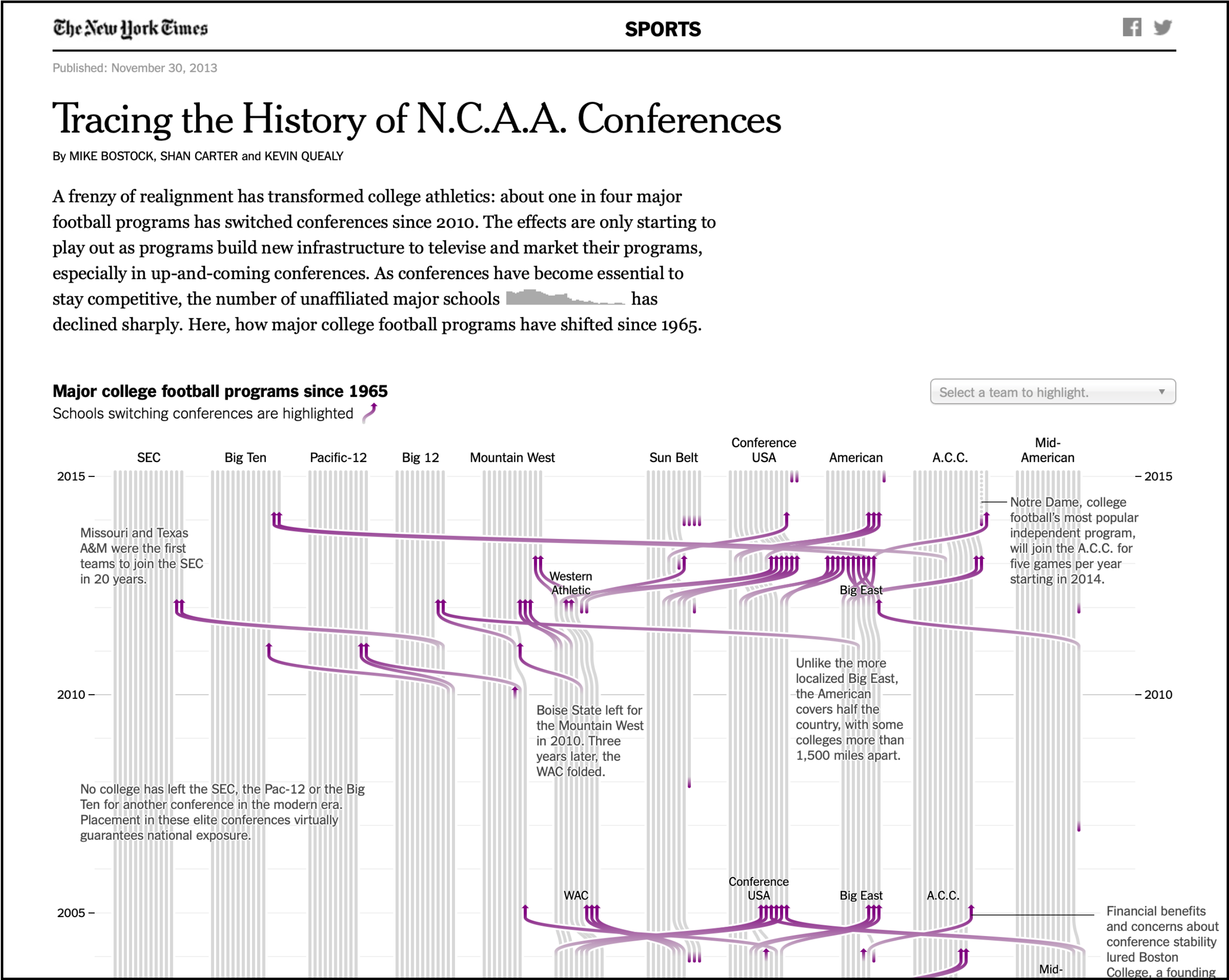
**Move from
exploring to refining.**

Doumont applied to data encoding, which works best? — iterative process of creating, questioning, testing!



Doumont applied to data encoding, which works best? — *iterative process of creating, questioning, testing!*



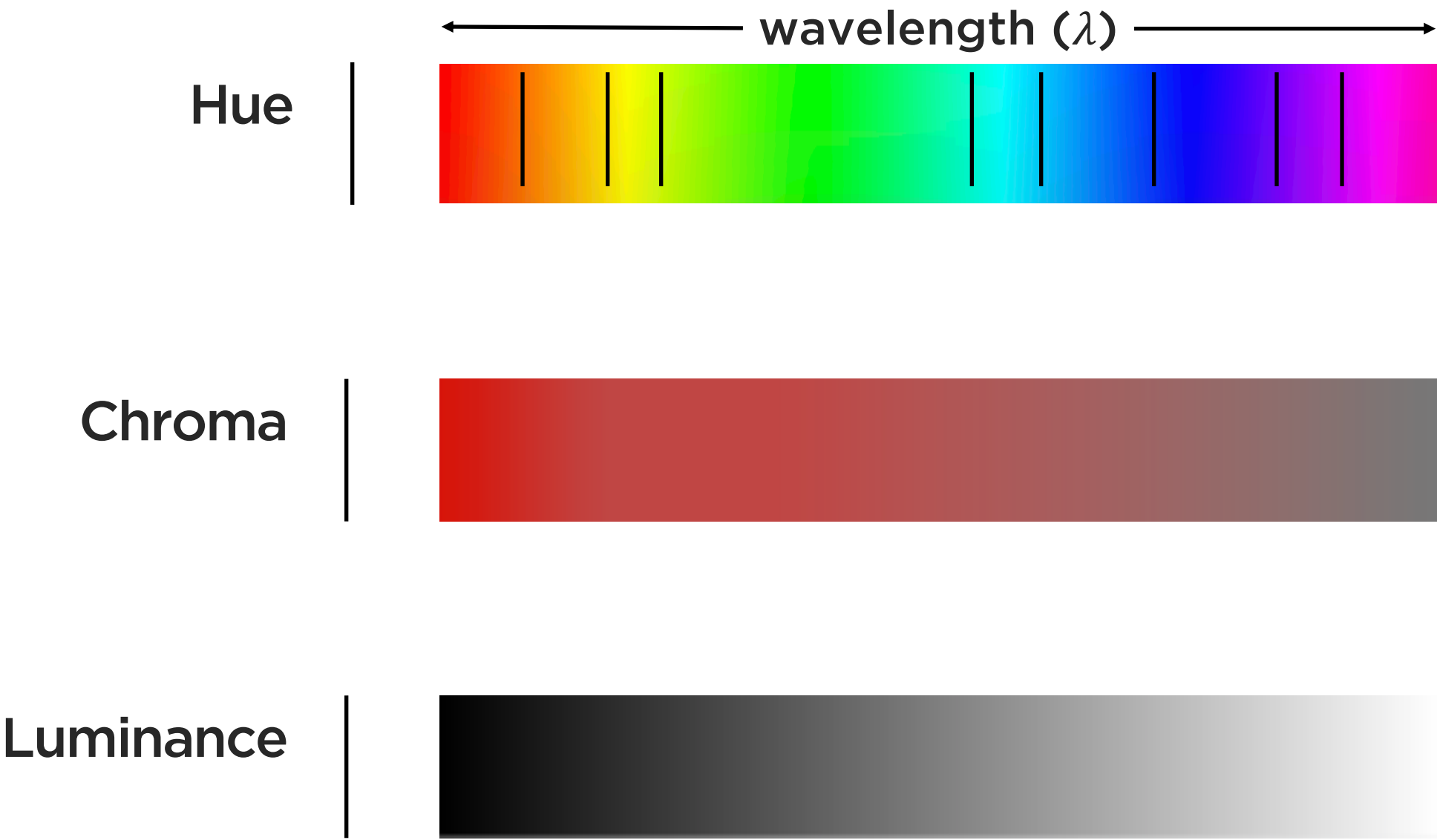
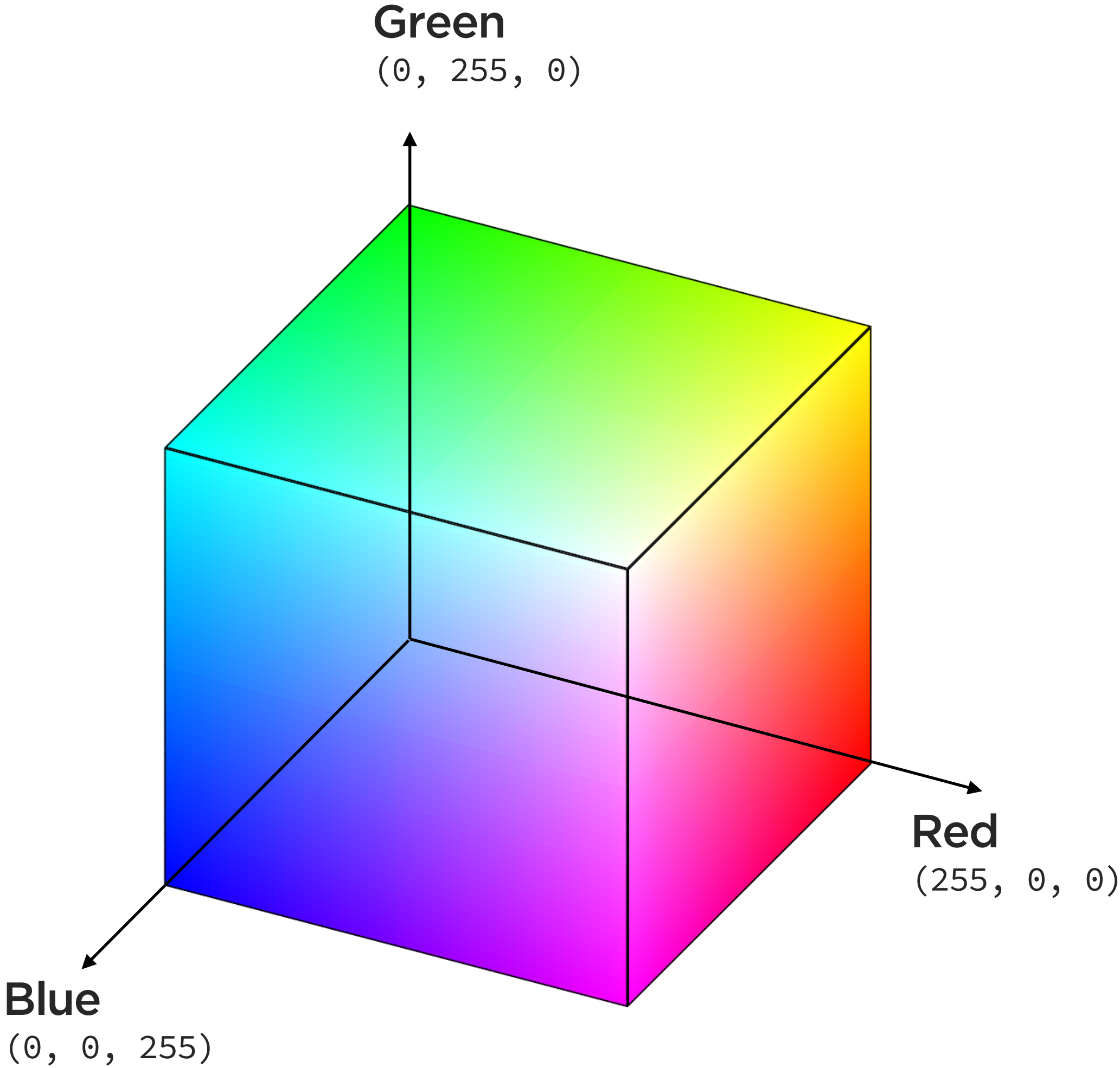


Doumont applied to data encoding, which works best? — *iterative process of creating, questioning, testing!*



encoding data as color

encoding data as color, encode data using color spaces, which are mathematical models



encoding data as color, how can we map data to light, whether using its hue, chroma, or luminance?

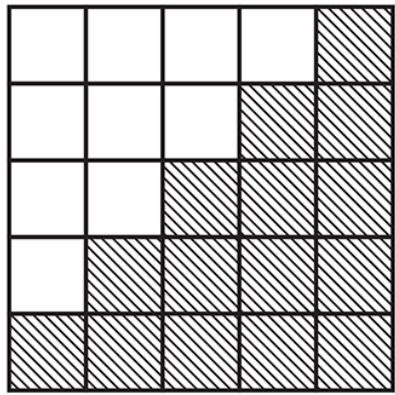


encoding data as color, **perceived brightness is nonlinear function of luminance**

LUMINANCE : the *measured* amount of light coming from some region of space.

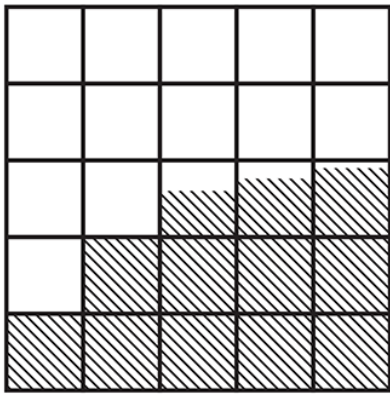
BRIGHTNESS : the *perceived* amount of light coming from that region of space.

encoding data as color, visual perception of arithmetical progression depends on physical geometric progression

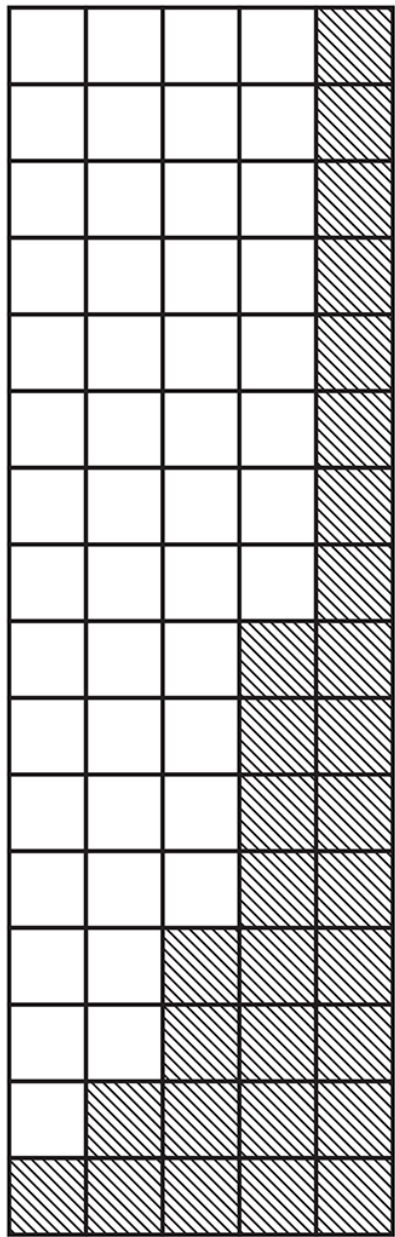


THIS PHYSICAL FACT

REDUCES TO

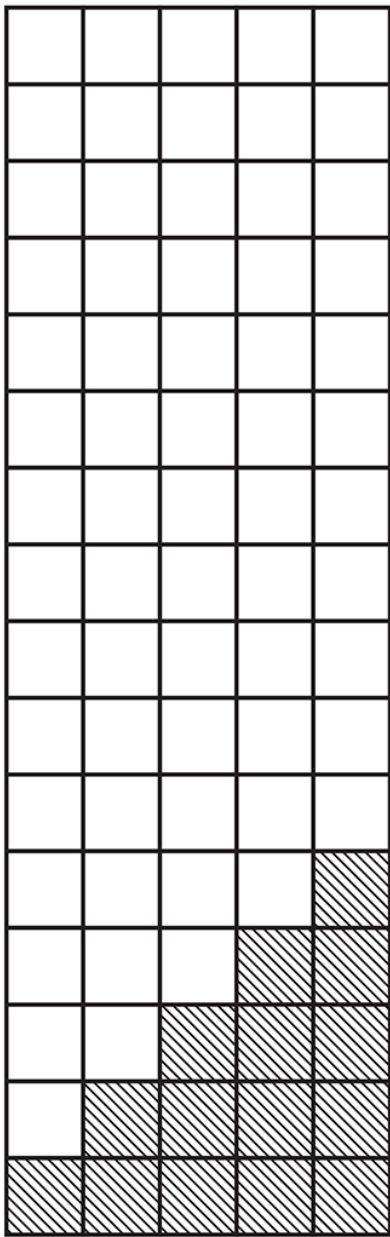


THIS PSYCHOLOGICAL EFFECT



THIS PHYSICAL FACT

PRODUCES



THIS PSYCHOLOGICAL EFFECT

color, HSL colorspace is intuitive, but not perceptually uniform in each attribute

Same luminance or lightness?

HSL(250, 100, 100)

HSL(250, 100, 100)

HSL(60, 100, 100)

HSL(60, 100, 100)

color, HSL colorspace is intuitive, but not perceptually uniform in each attribute

Same saturation?

HSL(0, 30, 40)

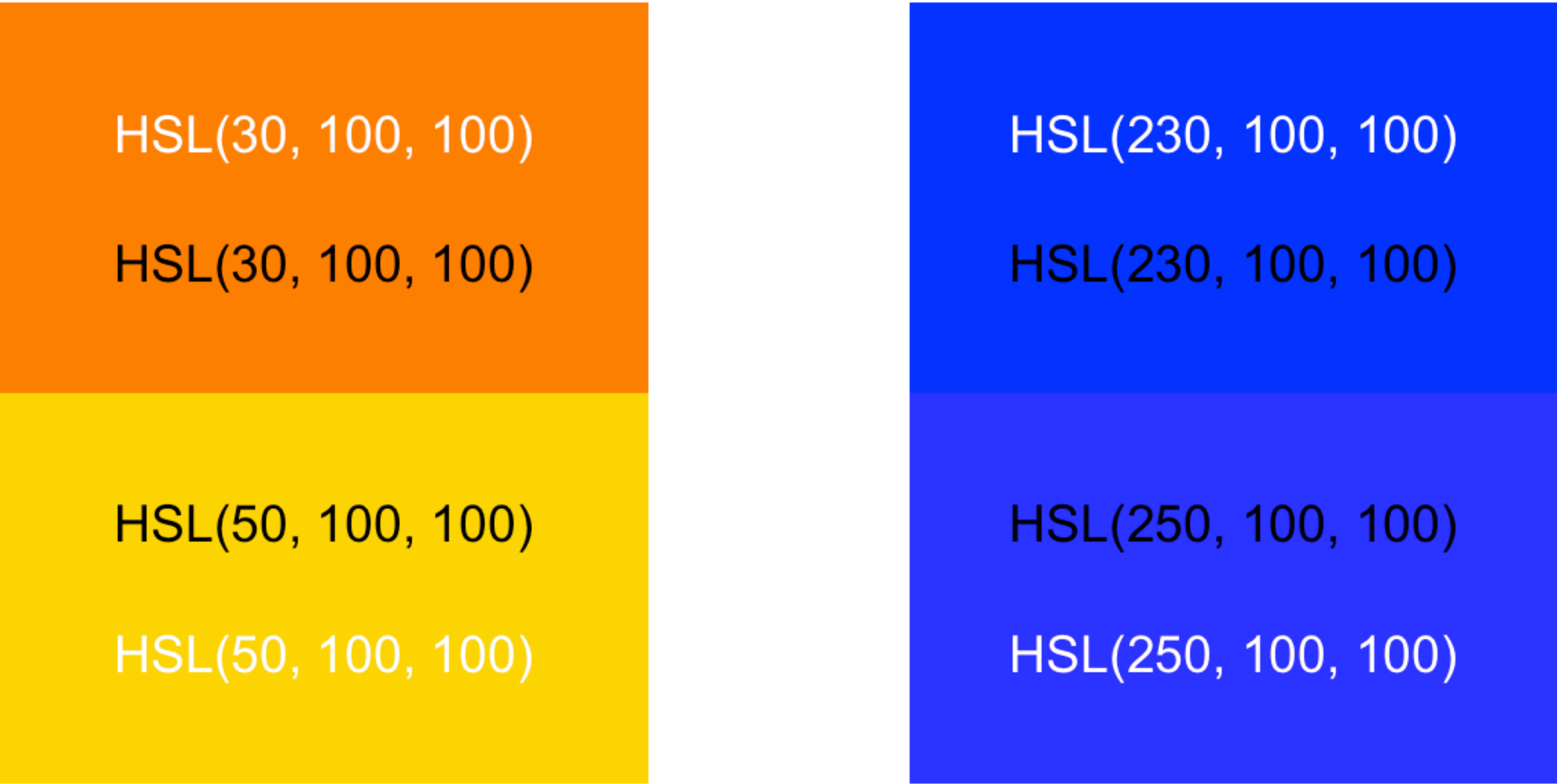
HSL(0, 30, 40)

HSL(0, 30, 90)

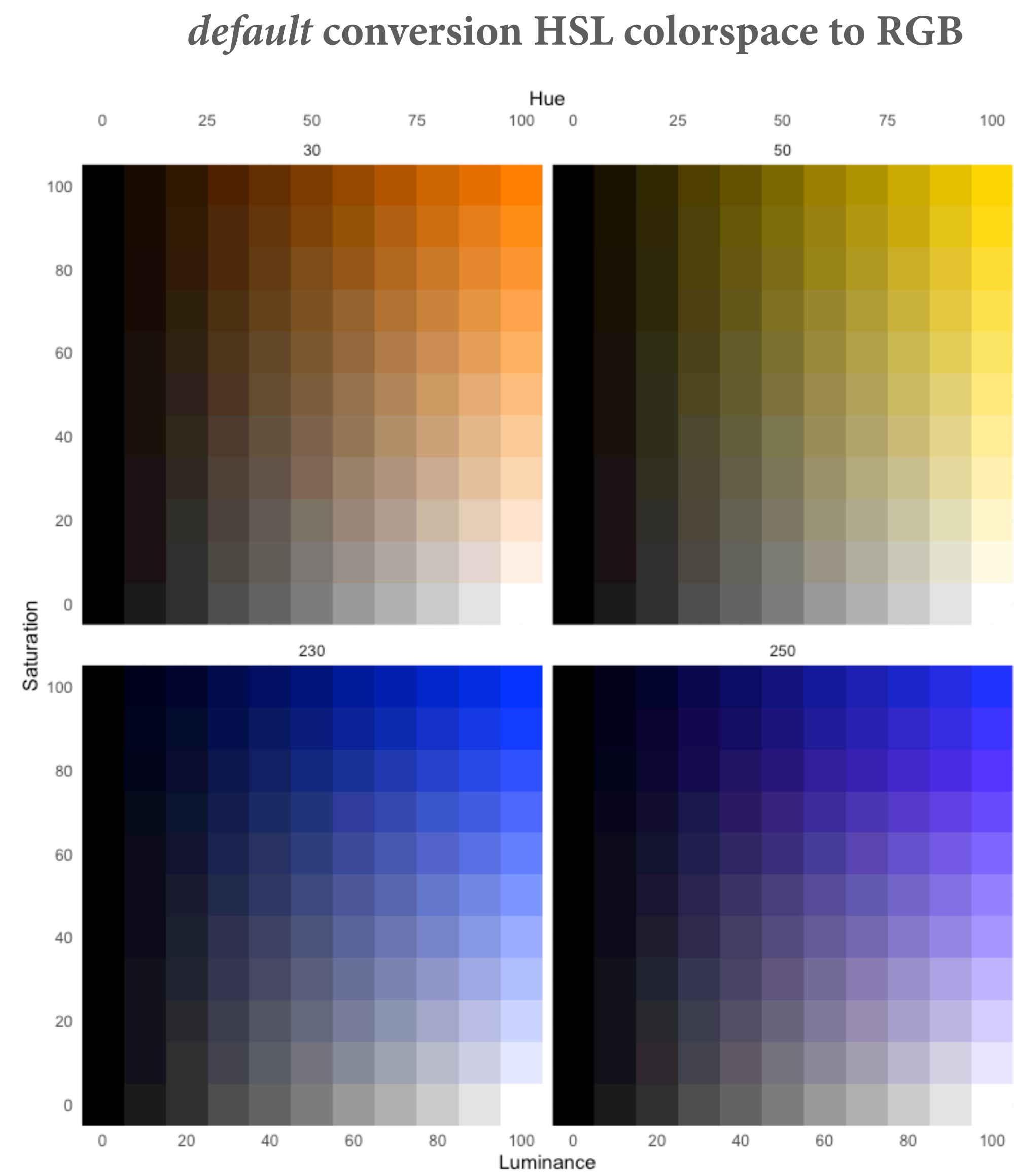
HSL(0, 30, 90)

color, HSL colorspace is intuitive, but not perceptually uniform in each attribute

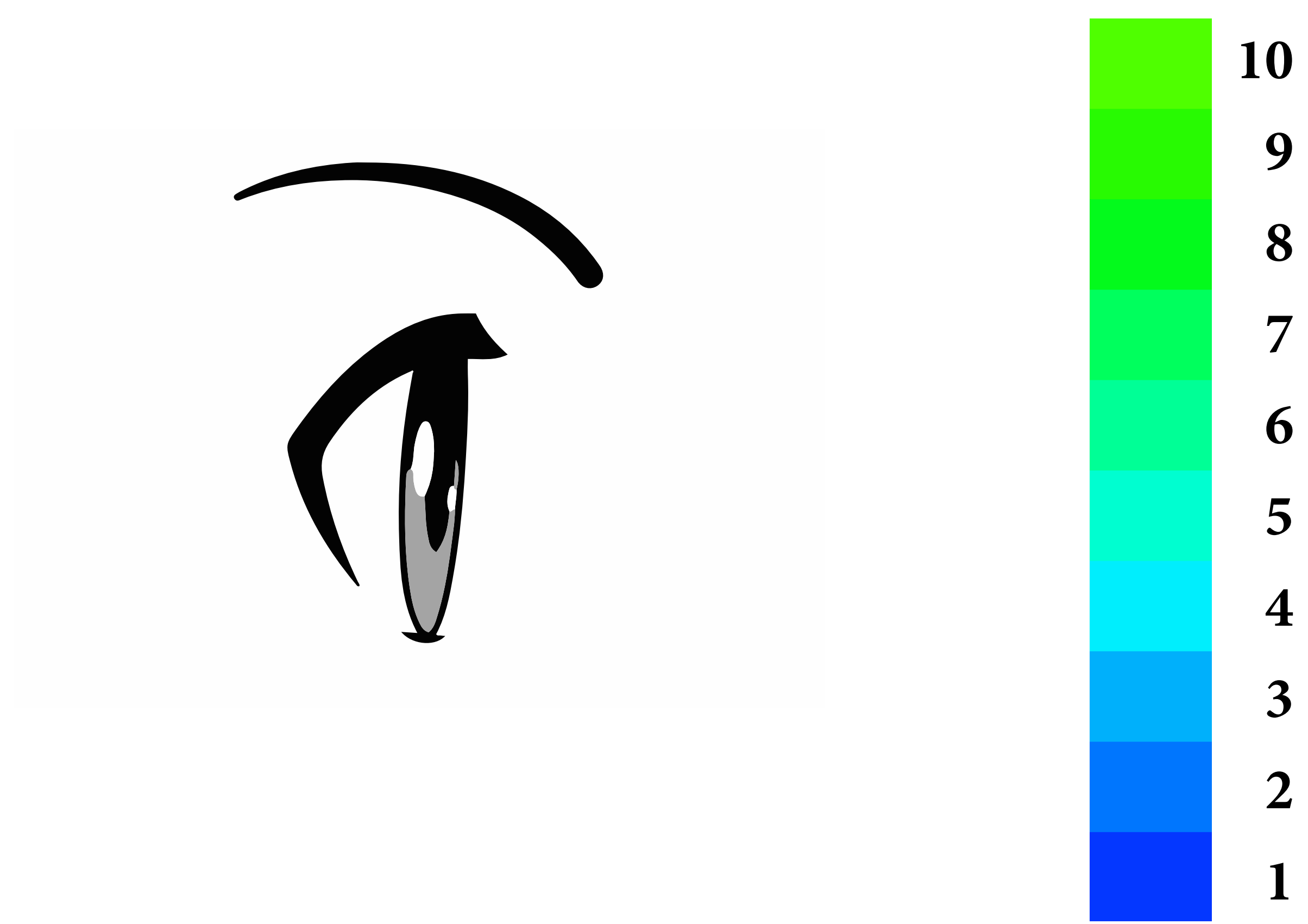
Equal difference between hues?



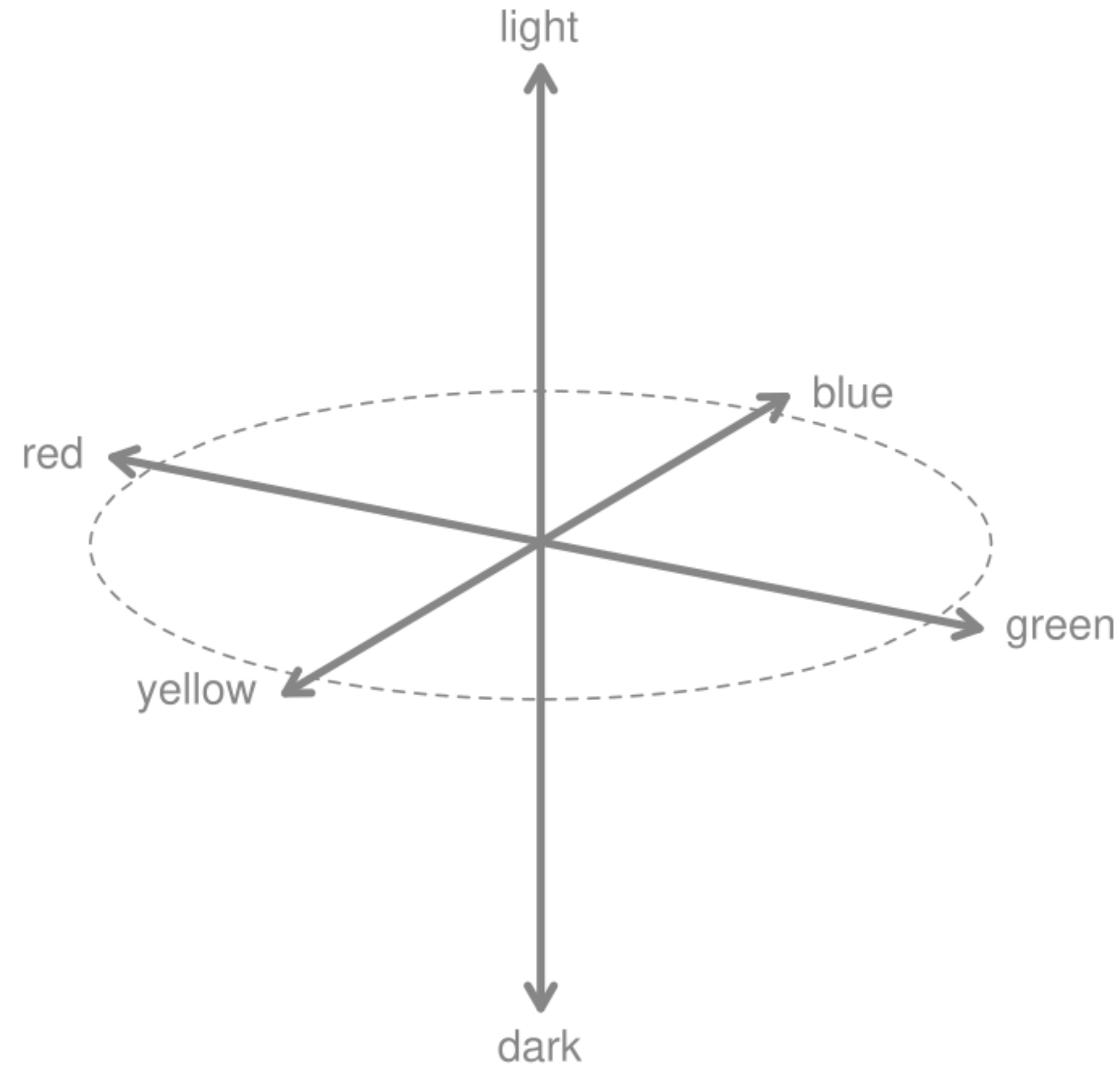
color, example encoding data into hue, saturation, and luminance



color, as with luminance, *hue* values in the RGB color space fail to uniformly scale across values



color, perceptually uniform color spaces have been created — CIELuv — but it isn't intuitive like HSL



The International Commission on Illumination (CIE) studied human perception and re-mapped color into a space where we perceive color changes uniformly.

Their **CIELuv** color model has two dimensions — *u* and *v* — that represent color scales from red to green and yellow to blue.

color, example encoding data as *perceptually uniform* color attributes: R · ggplot2 · HSLuv

Load functions for mapping data to perceptually-uniform color, from my R package: <https://github.com/ssp3nc3r/hsluv-rcpp>

```
library(HSLuv)
```

Create sample data encoded as hue, saturation, luminance

```
df <- expand.grid(H = c(30, 50, 230, 250),  
                 S = seq(0, 100, by = 10),  
                 L = seq(0, 100, by = 10))
```

Map or rescale your data values to valid range for the given the visual channel. This example data are already scaled to HSL ranges, so we don't need to rescale:

```
library(scales)
```

```
df <- df %>%  
  mutate(H = rescale(H, from = c(0, 360), to = c(0, 360) ),  
         S = rescale(S, from = c(0, 100), to = c(0, 100) ),  
         L = rescale(L, from = c(0, 100), to = c(0, 100) ))
```

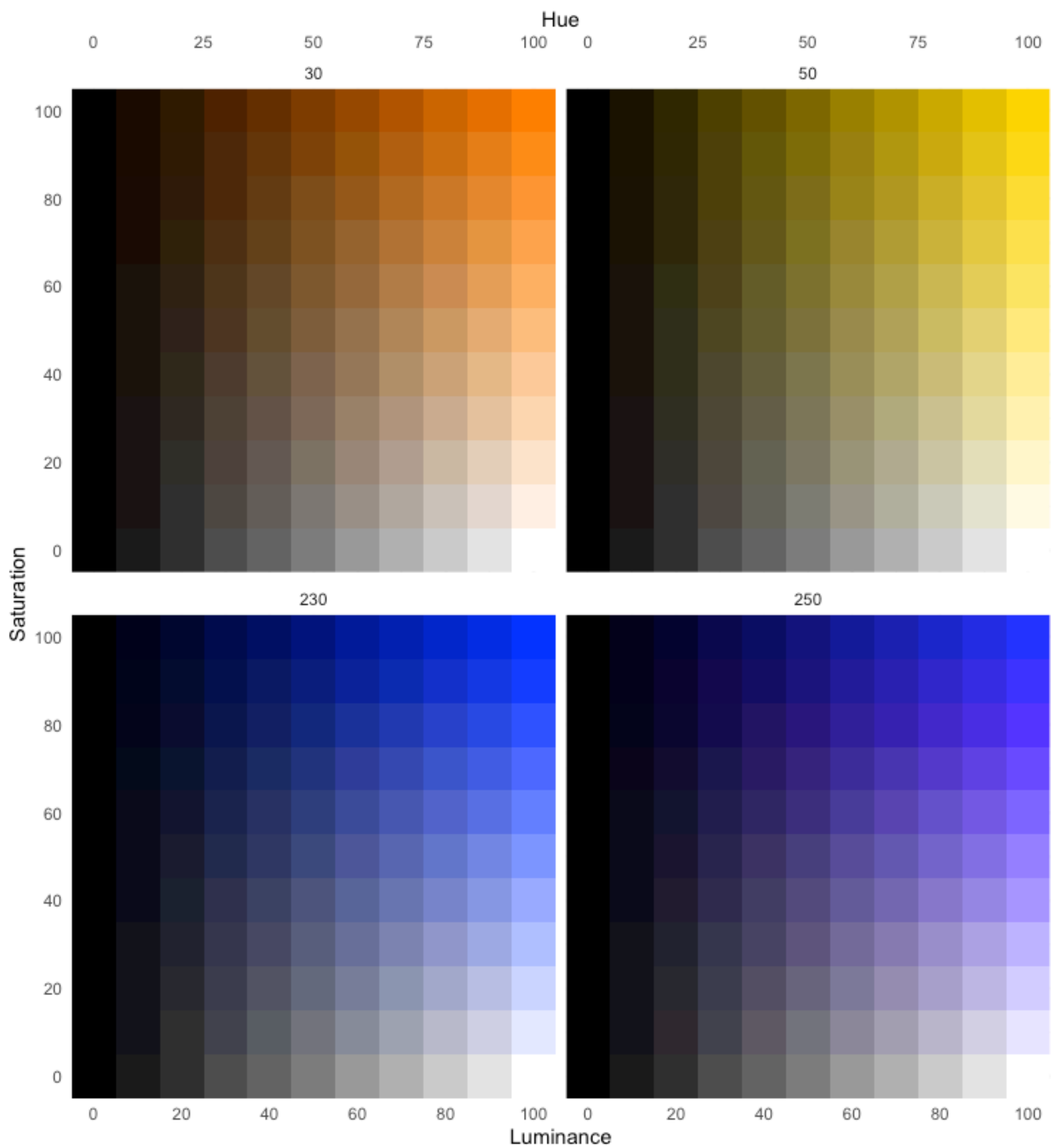
— data range — —visual range—

Plot data encoded as colors

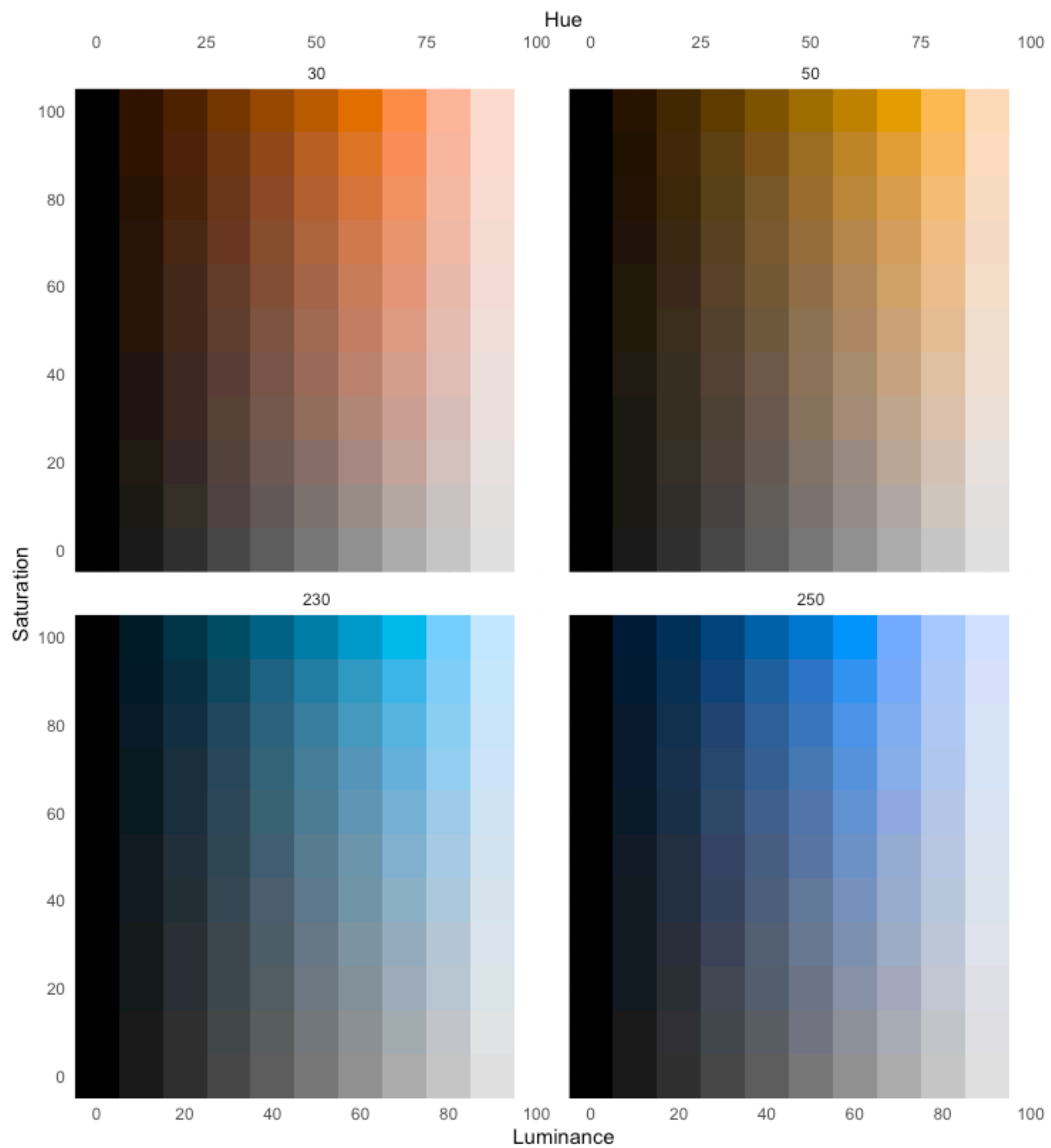
```
ggplot(df) +  
  
  facet_wrap(~ H ) +  
  
  scale_x_continuous(  
    name = 'Luminance',  
    breaks = seq(0, 100, by = 20),  
    expand = c(0,0),  
    sec.axis = sec_axis(~., name = 'Hue')) +  
  
  scale_y_continuous(  
    name = 'Saturation',  
    breaks = seq(0, 100, by = 20),  
    expand = c(0,0)) +  
  
  scale_fill_identity() +  
  
  geom_raster(  
    mapping = aes(  
      x = L,  
      y = S,  
      fill = hsluv_hex(H, S, L)),  
  )
```

color, example encoding data into hue, saturation, and luminance

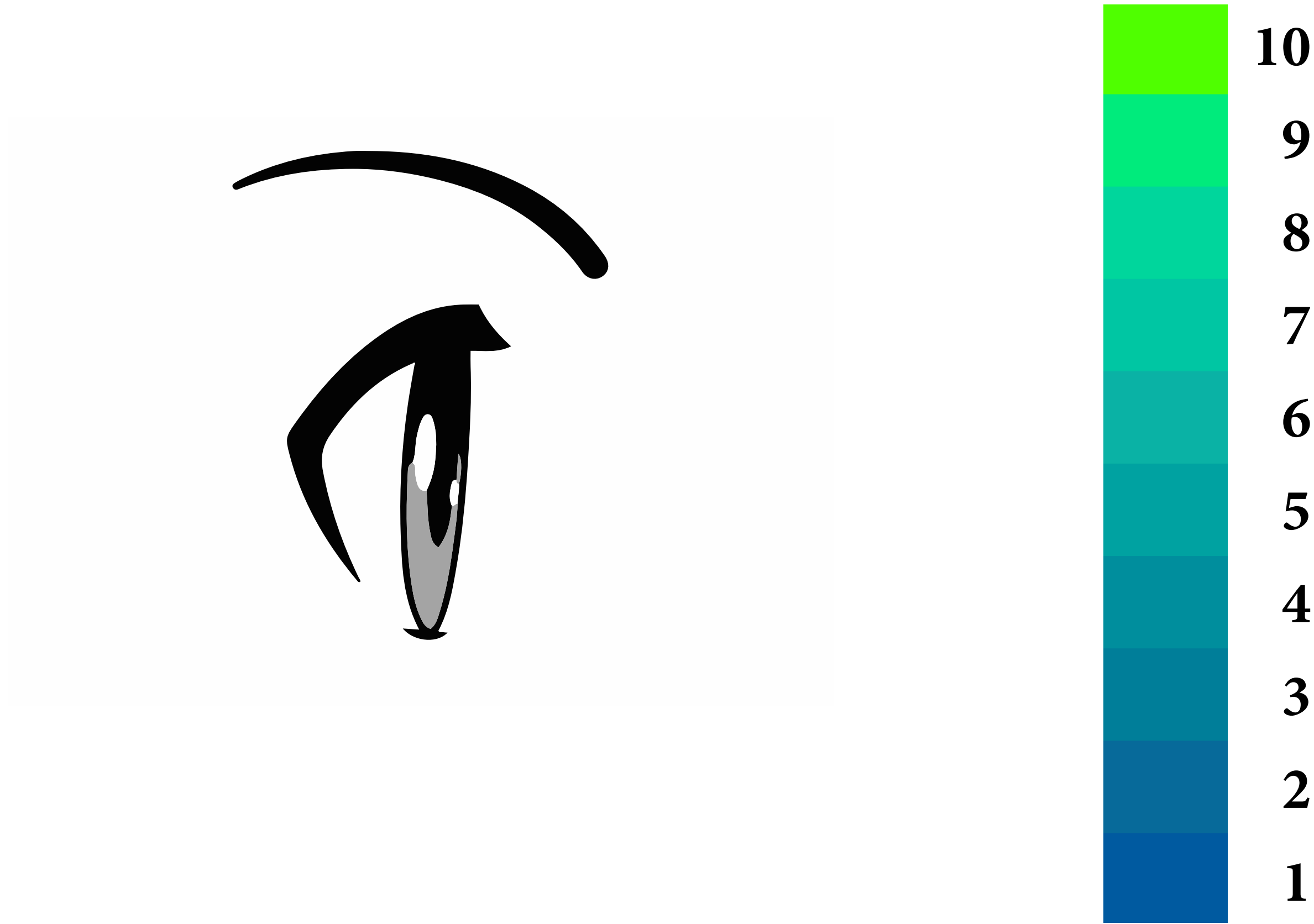
default conversion HSL colorspace to RGB



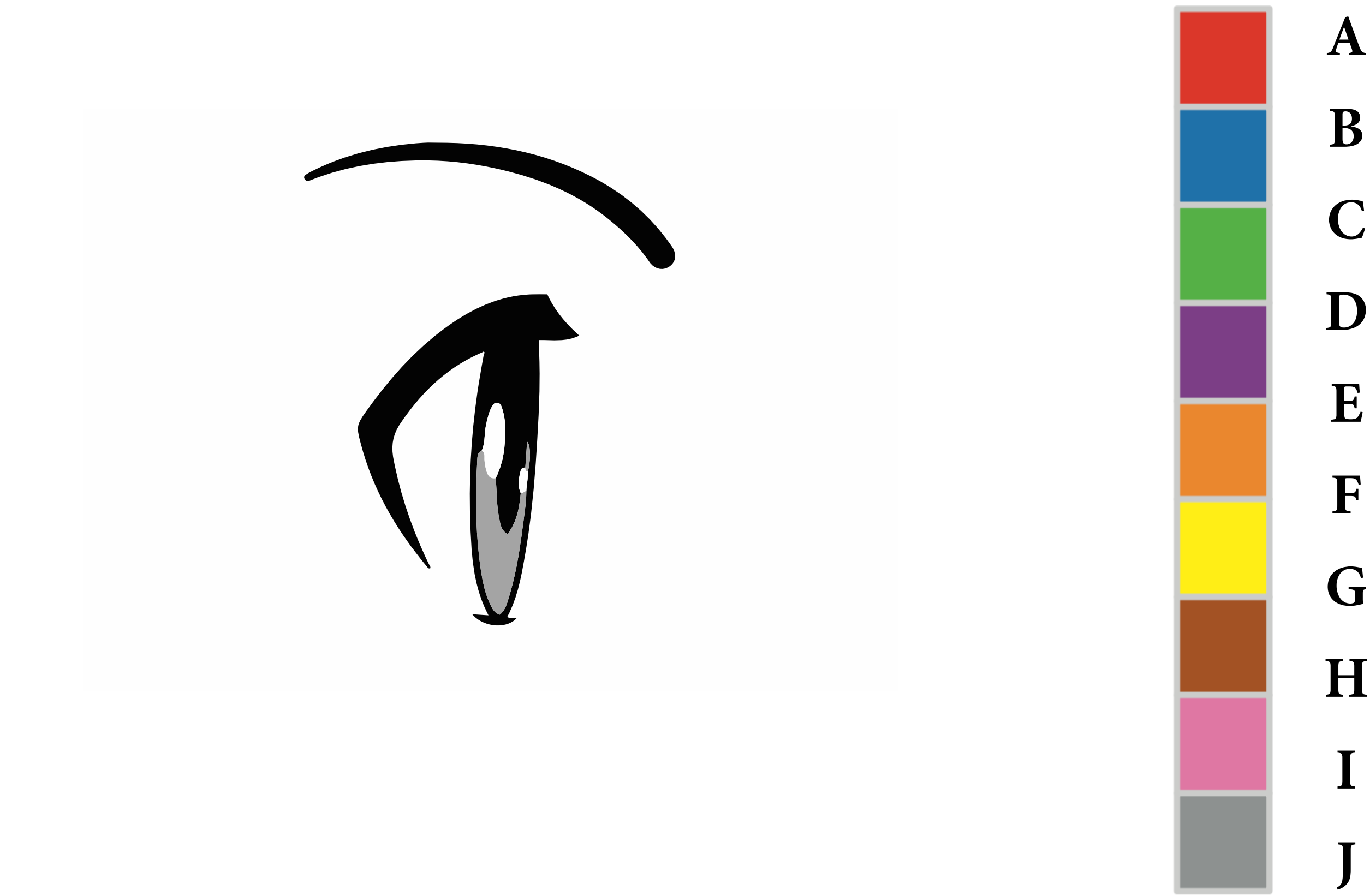
perceptually uniform conversion HSLuv colorspace to RGB



color, perceptually uniform color spaces better represent quantity

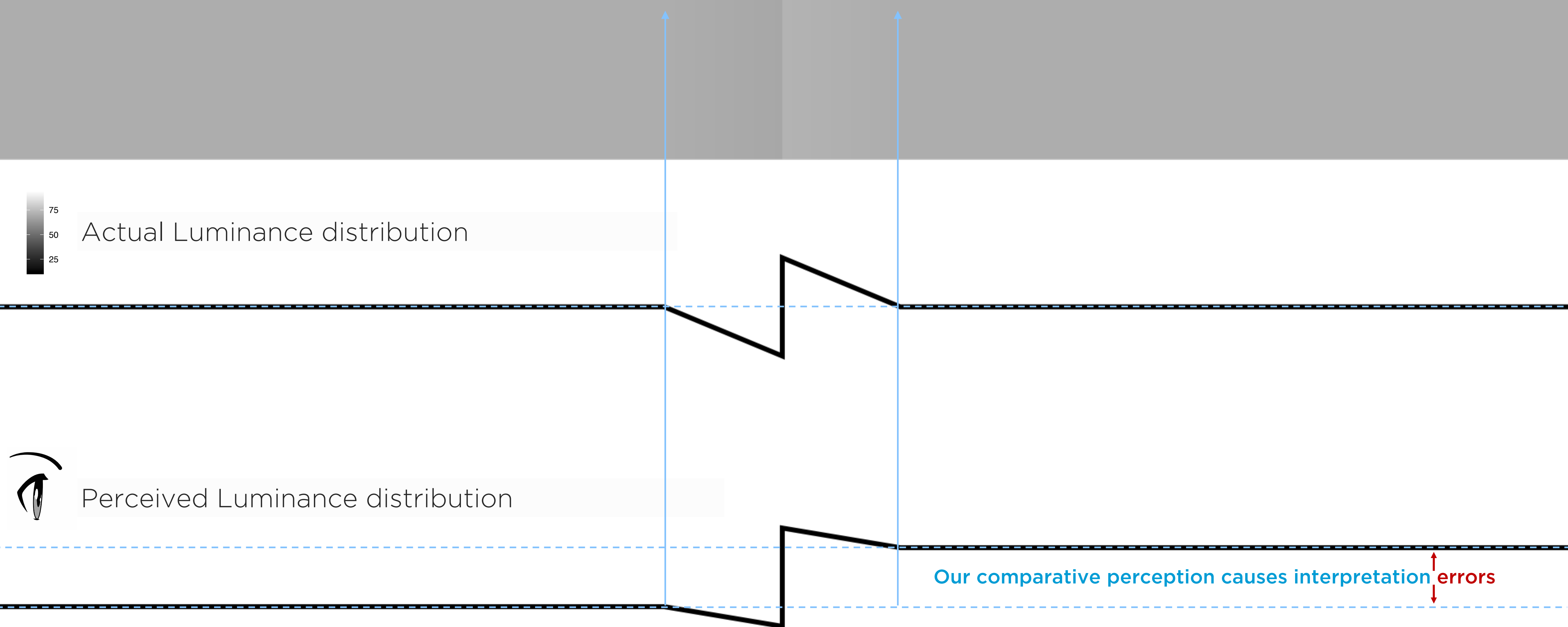


color, perceptually uniform color spaces also help in distinguishing categorical data

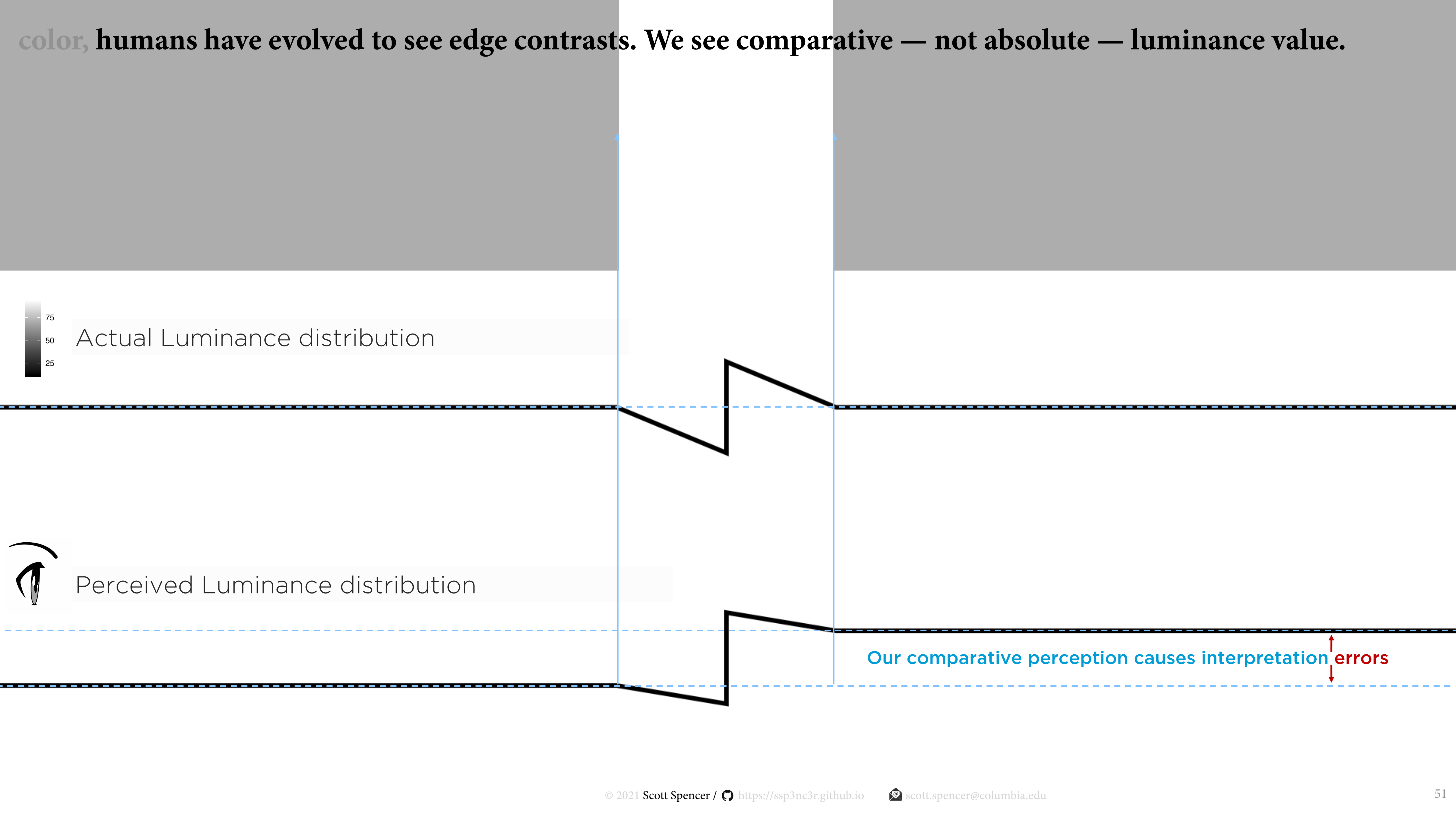


interaction of color

color, humans have evolved to see edge contrasts. We see comparative — not absolute — luminance value.



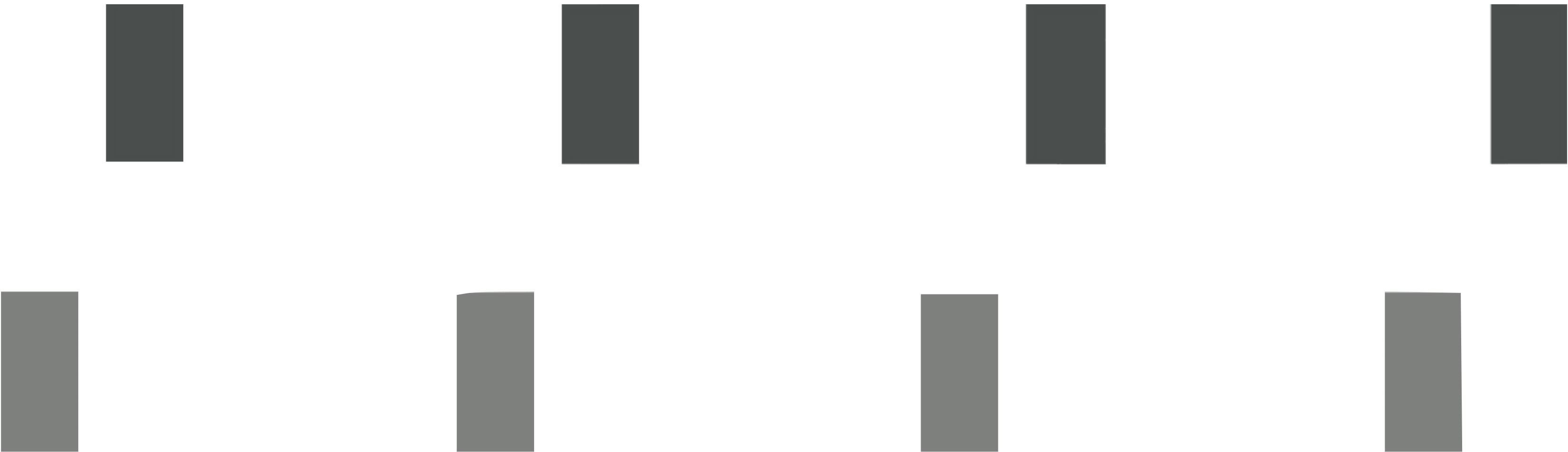
color, humans have evolved to see edge contrasts. We see comparative — not absolute — luminance value.



color, background and adjacent luminance can interfere with our perception



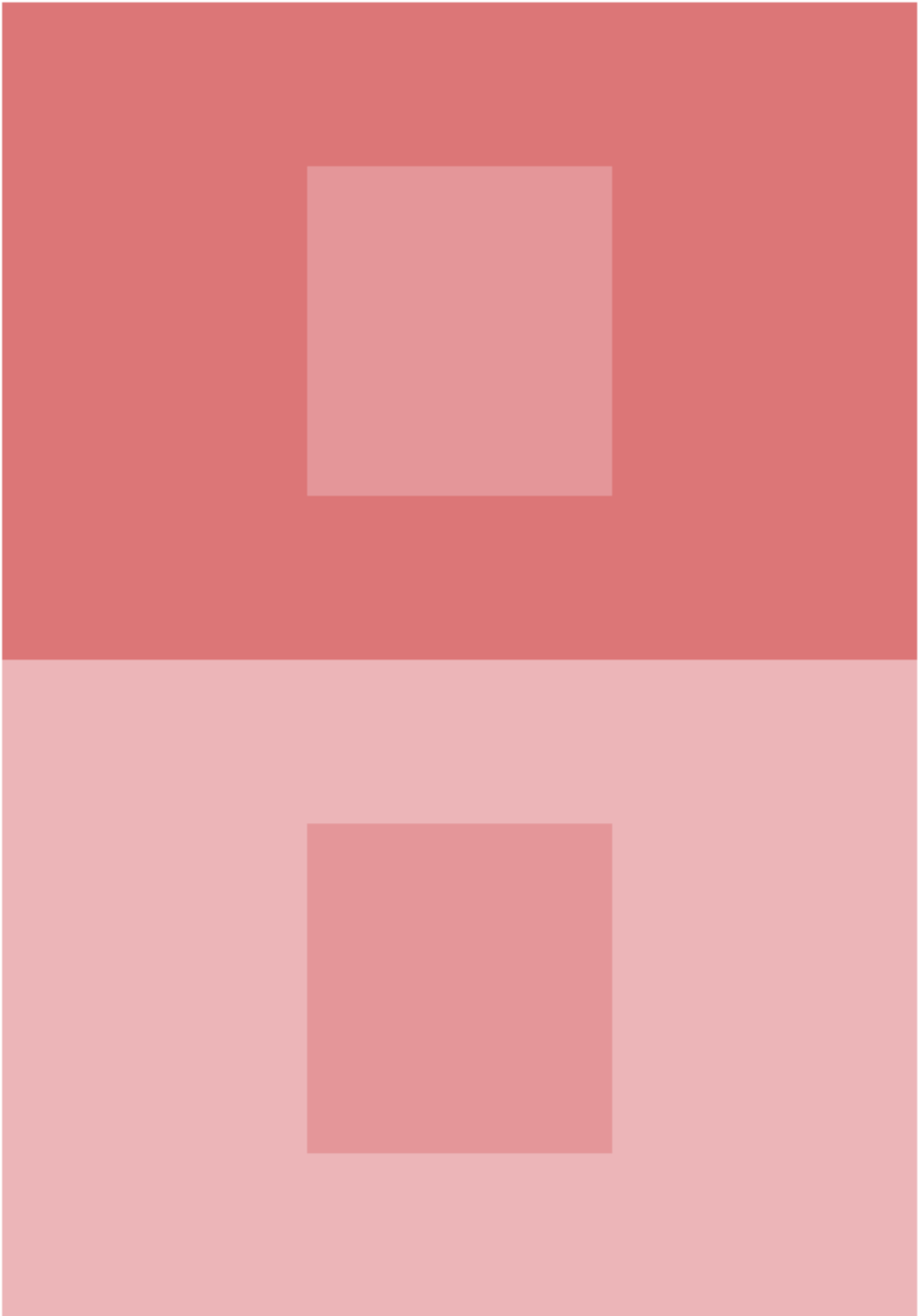
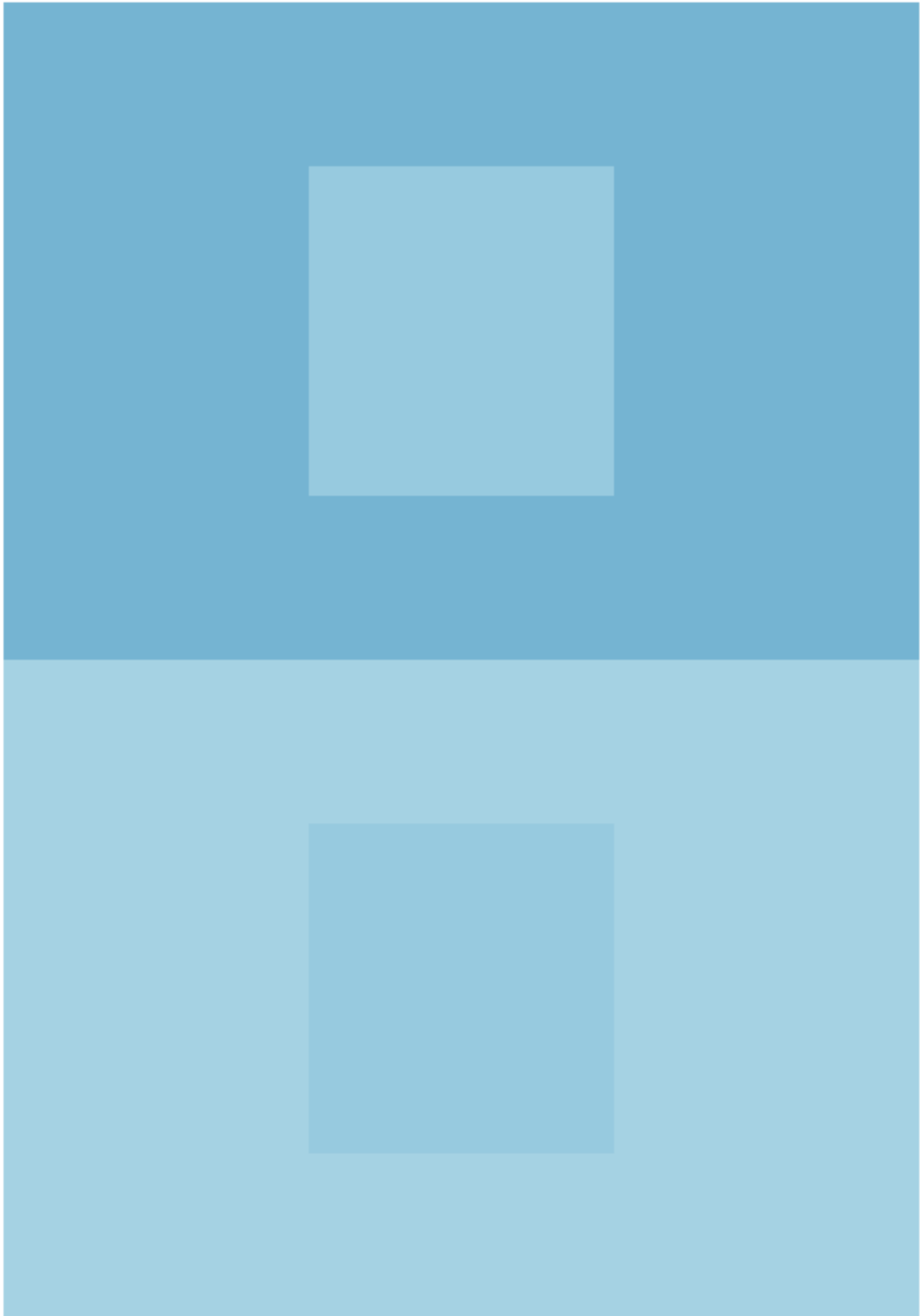
color, background and adjacent luminance can interfere with our perception



color, high foreground to background luminance contrast enhances shape, lower contrast enhances grayscale



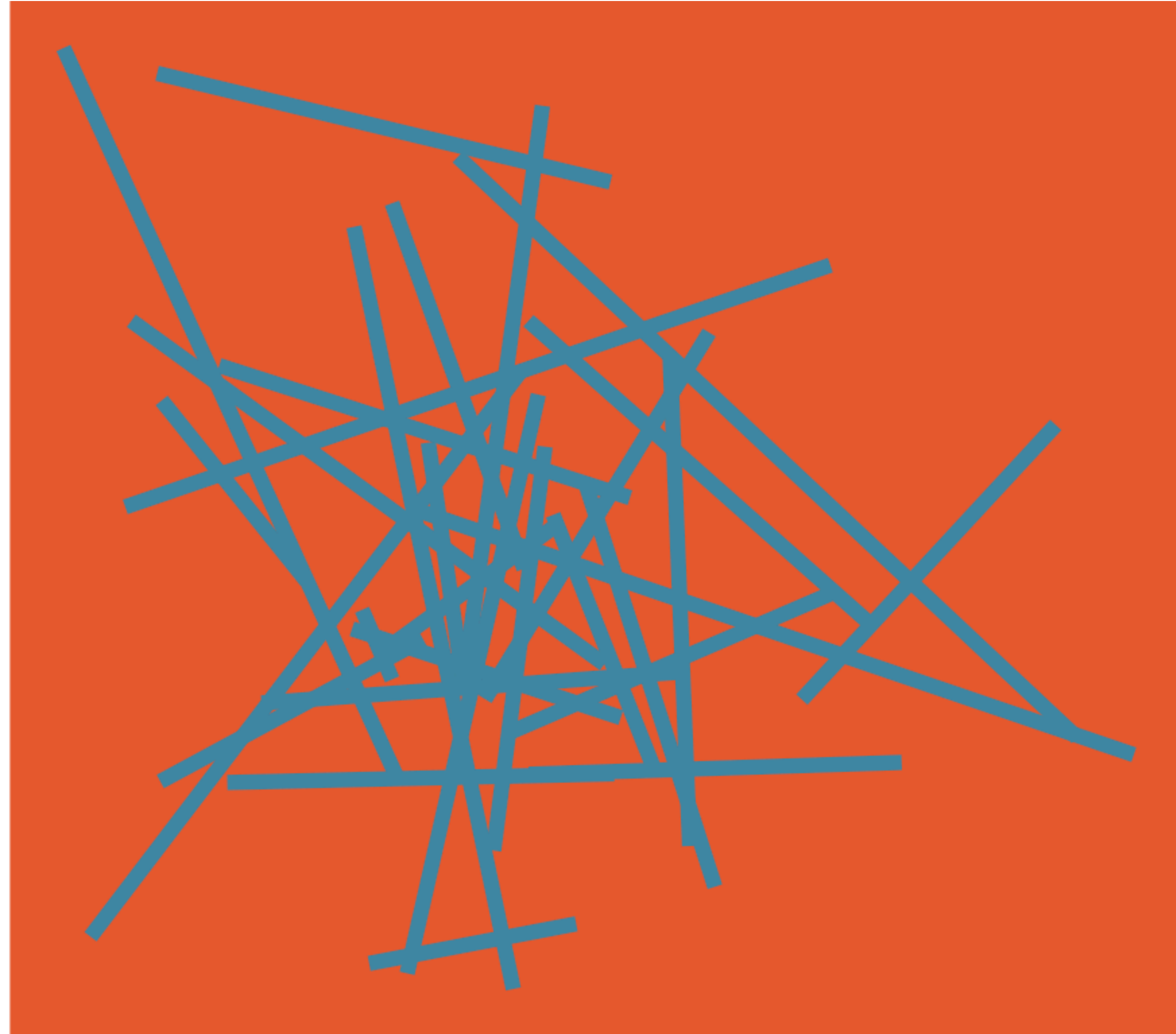
interaction of color, one color appearing as two



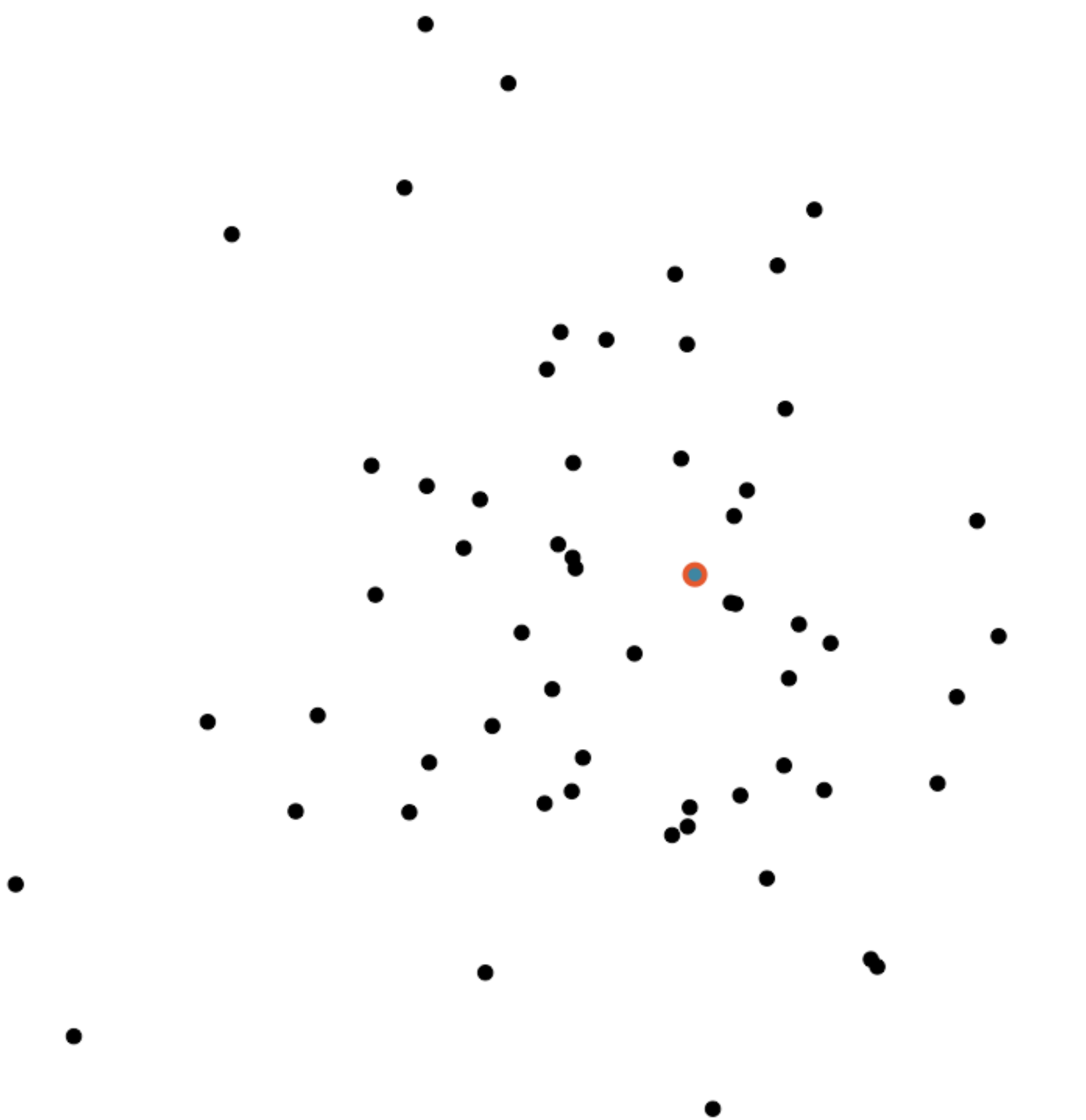
interaction of color, two different colors look alike



interaction of color, vibrating boundaries, occurs with contrasting hues of similar luminance



interaction of color, experimentation with vibrating boundaries to focus attention



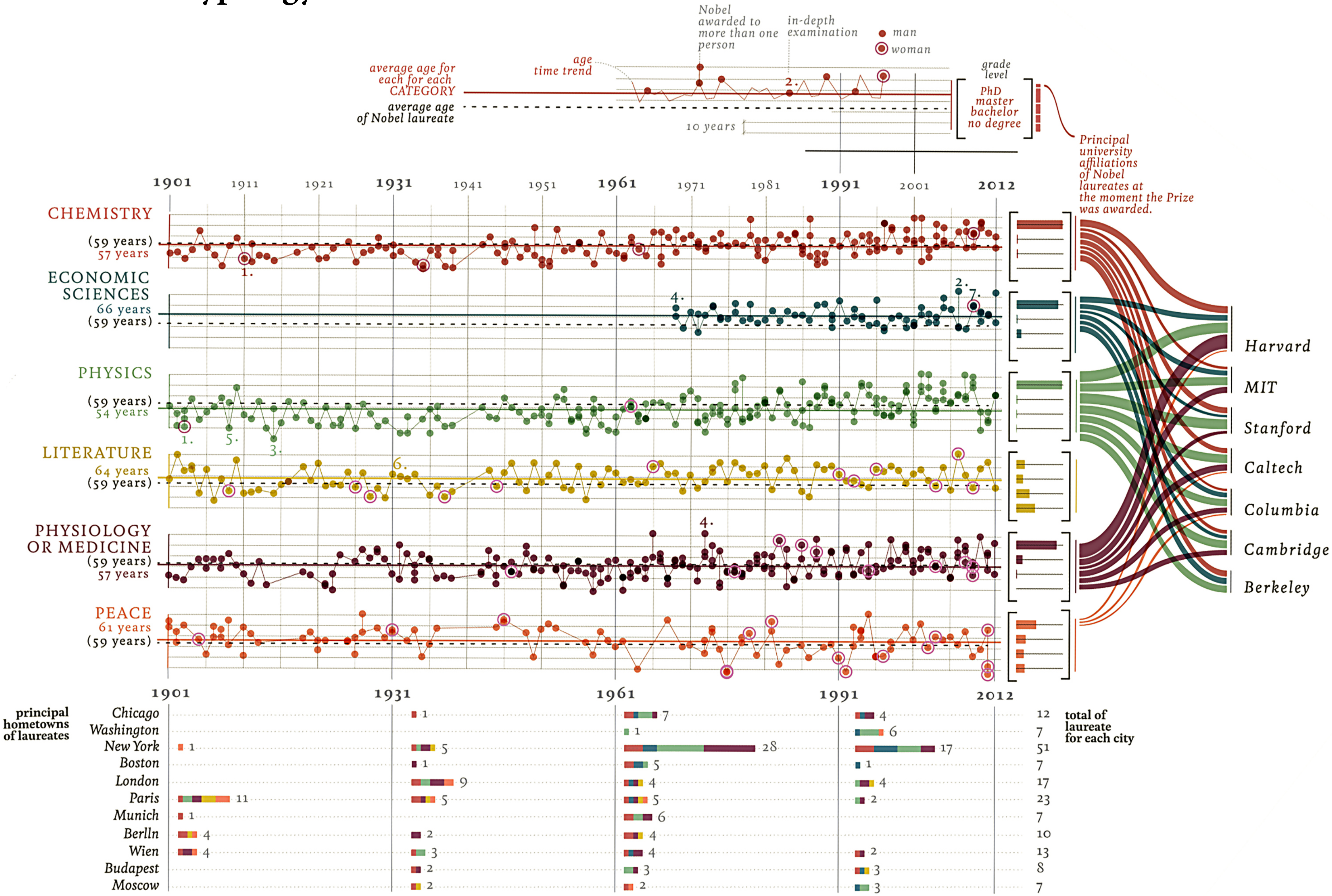
**a graphics study — deconstructing Lupi's
Nobels, no degrees, identifying typologies**

a graphics study through deconstruction and typology

If at first, this seems complex, Lupi’s graphic is just organized groups of layered data encodings. These even follow typologies commonly used in business communications. We can make something complex like this by creating component parts and carefully arranging them.

Don’t be intimidated! — Just methodically experiment with encodings for each data type, then organize them.

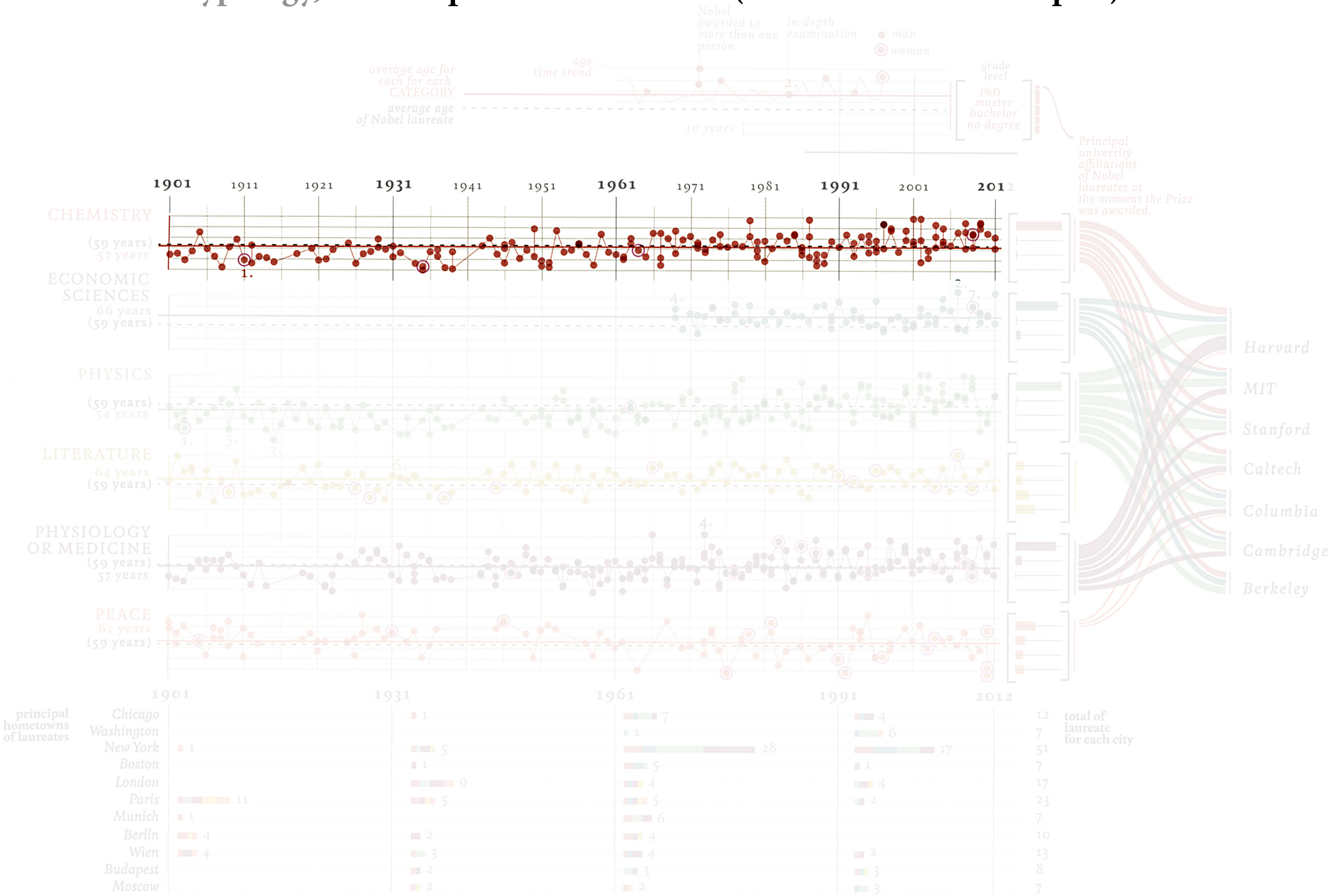
Of note: in Lupi’s organization, she aligns graphics by common axis scales. We’ll discuss this idea more later.



Lupi, Giorgia. 2016. *Visual Data - La Lettura*. <http://giorgialupi.com/lalettura>

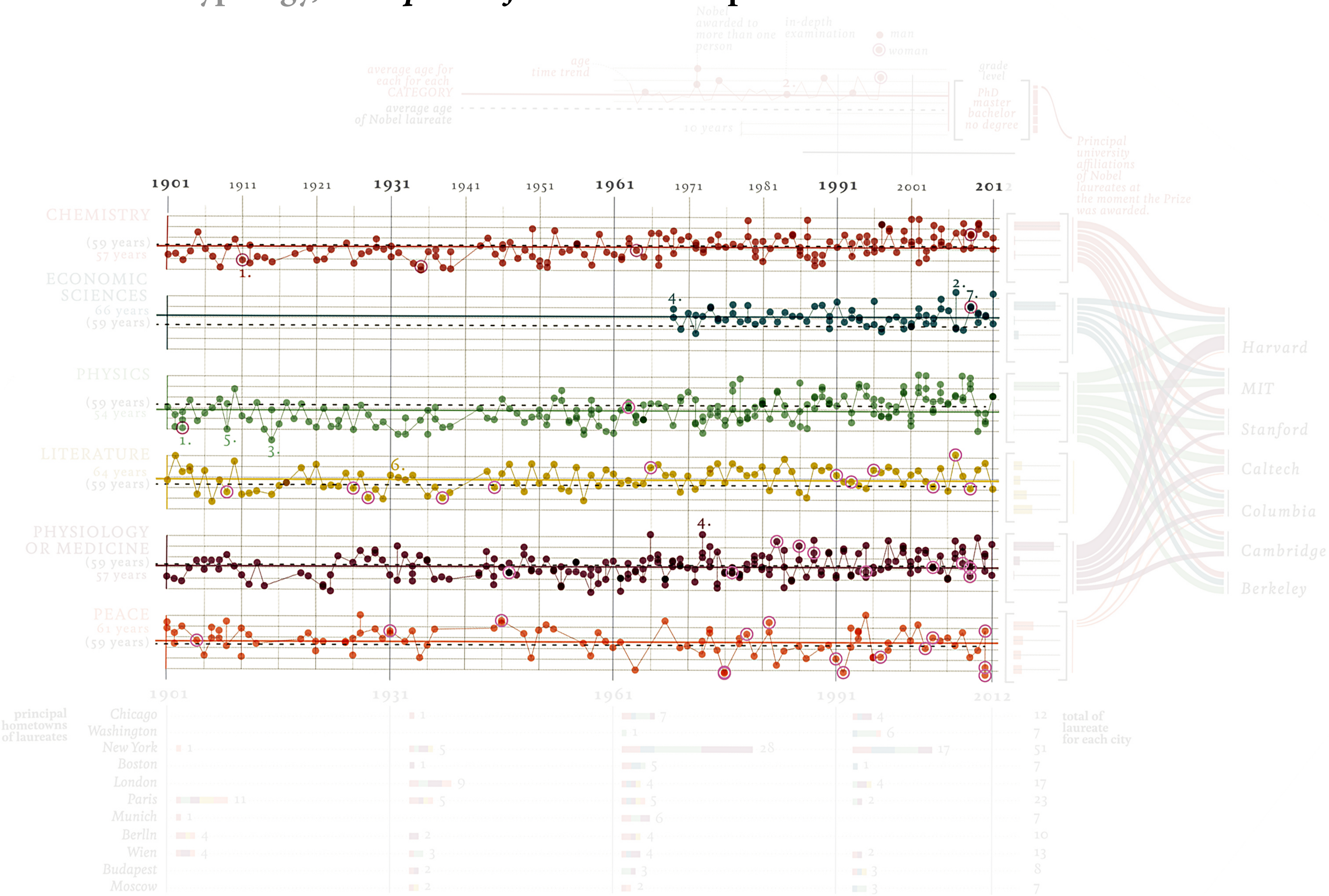
a graphics study through deconstruction and typology, a scatterplot and line charts (aka connected scatterplot)

```
ggplot(filter(df, Category == "Chemistry")) +  
  
  scale_color_manual(  
    values = category_colors,  
    breaks = category_names) +  
  
  scale_alpha_manual(  
    values = c(1, 0),  
    breaks = c("Female", "Male")) +  
  
  scale_x_continuous(  
    breaks = seq(1901, 2016, by = 30),  
    minor_breaks = seq(1901, 2016, by = 10),  
    position = "top") +  
  
  geom_hline(  
    mapping = aes(  
      yintercept = mean(Age, na.rm = TRUE)),  
    lwd = 0.2,  
    color = "black",  
    linetype = "dashed") +  
  
  geom_hline(  
    mapping = aes(  
      yintercept = cat_avg_age,  
      color = Category) ) +  
  
  geom_line(  
    mapping = aes(  
      x = Year,  
      y = Age,  
      color = Category),  
    lwd = 0.2) +  
  
  geom_point(  
    mapping = aes(  
      x = Year,  
      y = Age,  
      color = Category),  
    size = 1.5,  
    alpha = 0.5) +  
  
  geom_point(  
    mapping = aes(  
      x = Year,  
      y = Age,  
      alpha = Sex),  
    color = "pink",  
    shape = 21,  
    size = 4)
```



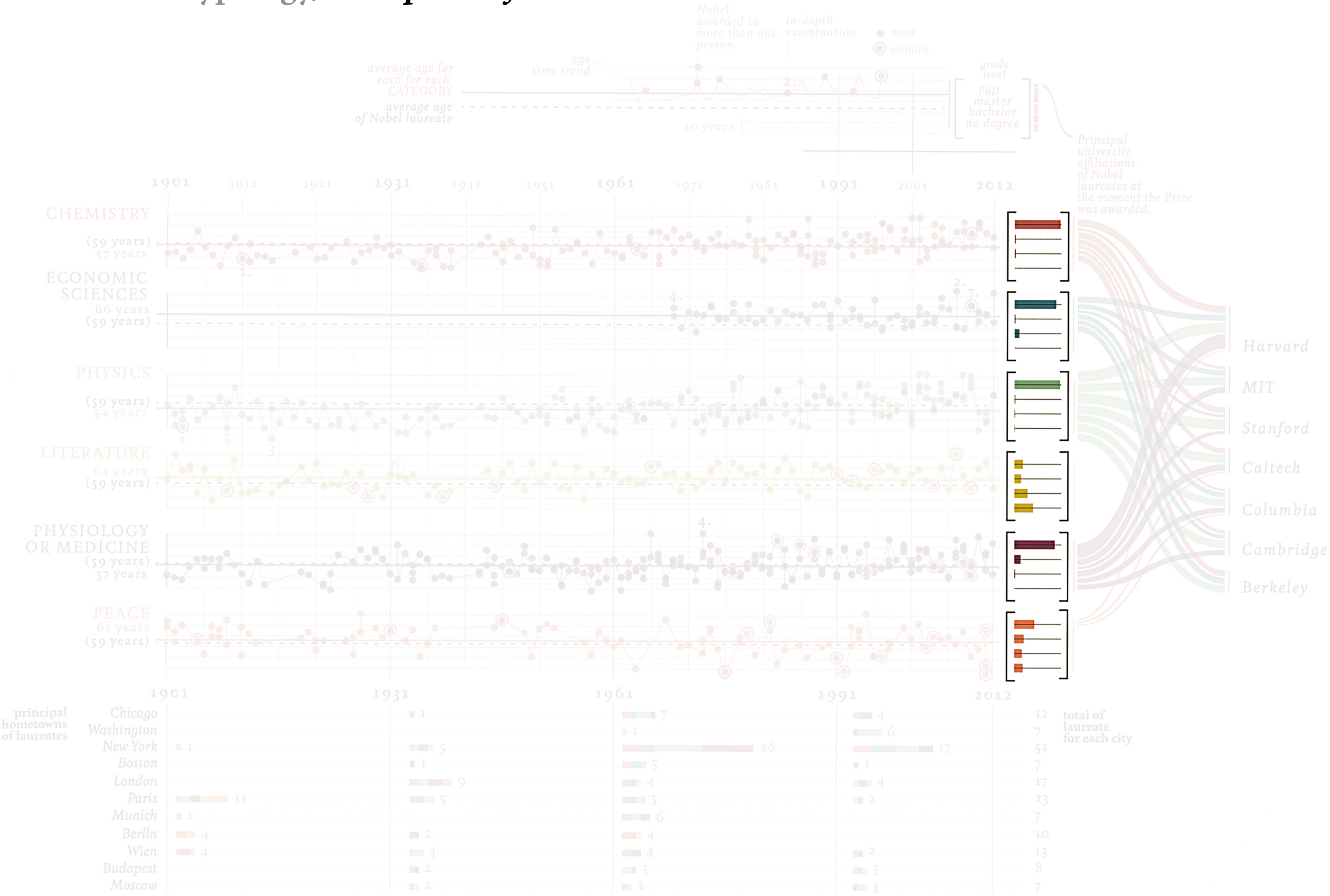
a graphics study through deconstruction and typology, *multiples or facets* of scatterplots and line charts

```
ggplot(df) +  
  
  facet_wrap(  
    Category ~ .,  
    nrow = 6,  
    strip.position = "left") +  
  
  scale_color_manual(  
    values = category_colors,  
    breaks = category_names) +  
  
  scale_alpha_manual(  
    values = c(1, 0),  
    breaks = c("Female", "Male")) +  
  
  scale_x_continuous(  
    breaks = seq(1901, 2016, by = 30),  
    minor breaks = seq(1901, 2016, by = 10),  
    position = "top") +  
  
  geom_hline(  
    mapping = aes(  
      yintercept = mean(Age, na.rm = TRUE)),  
    lwd = 0.2,  
    color = "black",  
    linetype = "dashed") +  
  
  geom_hline(  
    mapping = aes(  
      yintercept = cat_avg_age,  
      color = Category) ) +  
  
  geom_line(  
    mapping = aes(  
      x = Year,  
      y = Age,  
      color = Category),  
    lwd = 0.2) +  
  
  geom_point(  
    mapping = aes(  
      x = Year,  
      y = Age,  
      color = Category),  
    size = 1.5,  
    alpha = 0.5) +  
  
  geom_point(  
    mapping = aes(  
      x = Year,  
      y = Age,  
      alpha = Sex),  
    color = "pink",  
    shape = 21,  
    size = 4)
```



a graphics study through deconstruction and typology, *multiples or facets* of bar charts

```
ggplot(df) +  
  
  facet_wrap(  
    . ~ Category,  
    ncol = 1) +  
  
  scale_fill_manual(  
    values = category_colors,  
    breaks = category_names) +  
  
  geom_bar(  
    mapping = aes(  
      x = Percent,  
      y = Education,  
      fill = Category),  
    stat = "identity")
```



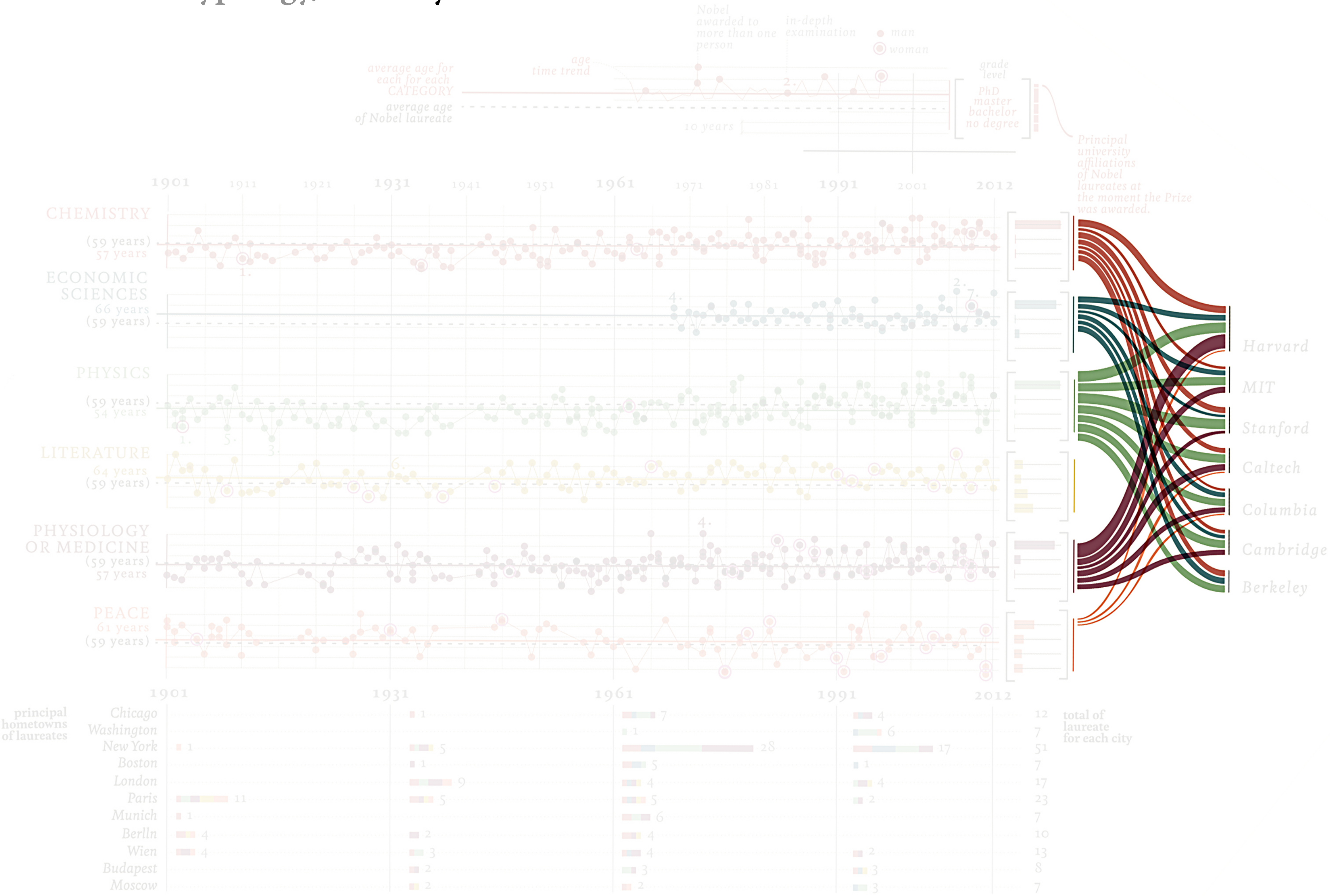
a graphics study through deconstruction and typology, *multiples or facets* of stacked bar charts

```
ggplot(df) +  
  
  facet_wrap(  
    era~.,  
    nrow = 1) +  
  
  scale_fill_manual(  
    values = category_colors,  
    breaks = category_names) +  
  
  geom_bar(  
    mapping = aes(  
      x = n,  
      y = Birth.City,  
      fill = Category),  
    stat = 'identity',  
    width = 0.2) +  
  
  geom_text(  
    mapping = aes(  
      x = n_era_city + 2,  
      y = Birth.City,  
      label = n_era_city),  
    stat = 'identity',  
    width = 0.2)
```



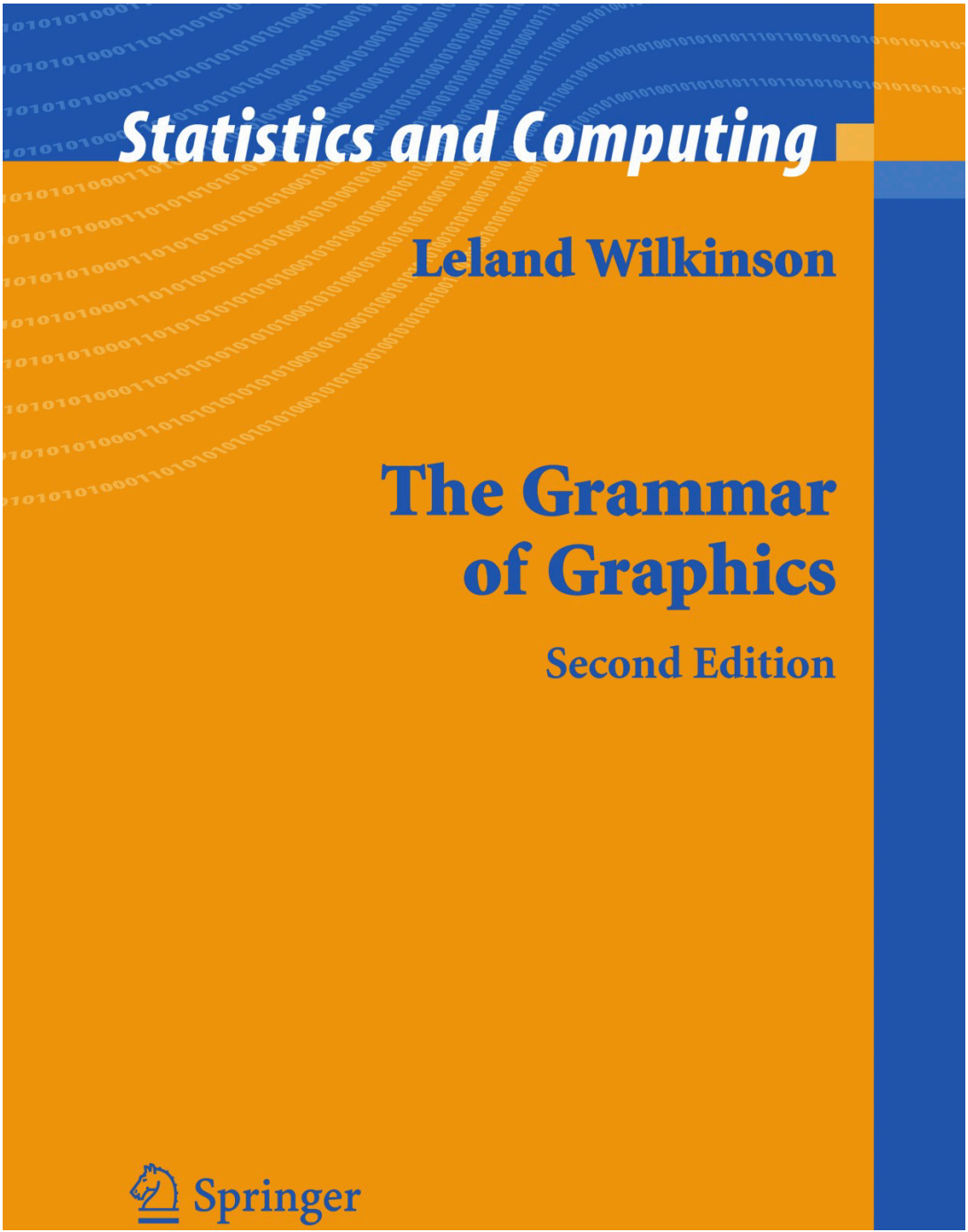
a graphics study through deconstruction and typology, a sankey chart

```
ggplot(  
  data = data,  
  mapping = aes(  
    x = x,  
    id = id,  
    split = y,  
    value = n)) +  
  
  scale_fill_manual(  
    values = category_colors,  
    breaks = category_names) +  
  
  geom_parallel_sets(  
    mapping = aes(  
      fill = Category),  
    alpha = 0.6,  
    axis.width = 0.05,  
    sep = 0.1) +  
  
  geom_parallel_sets_axes(  
    axis.width = 0.01,  
    fill = "gray80",  
    sep = 0.1)
```



charts are *mere typologies* of graphics — don't limit yourself

think data encodings, *not* charts ...

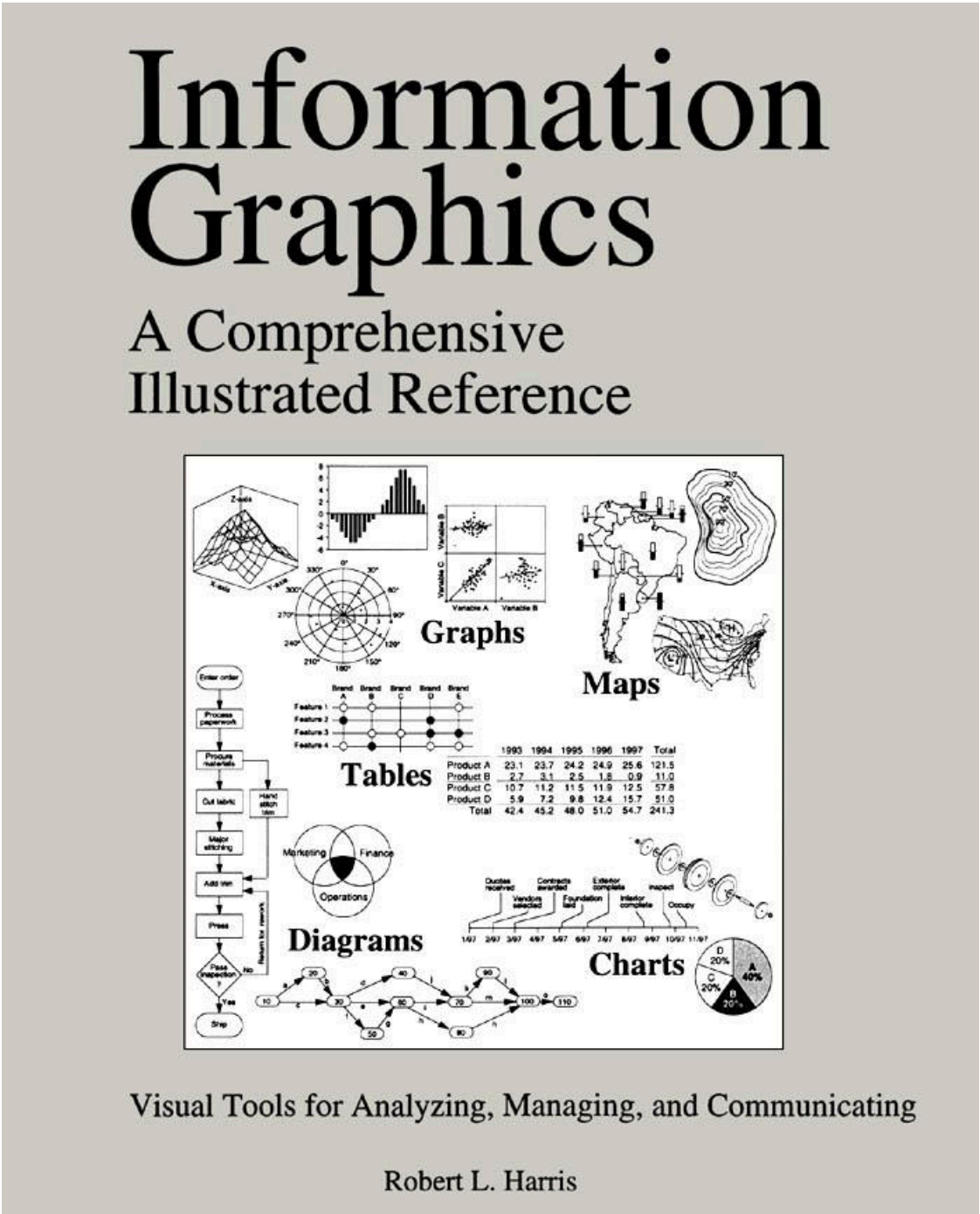


“We often call graphics charts. There are pie charts, bar charts, line charts, and so on. [We should] shun chart typologies. Charts are usually instances of much more general objects.

Once we understand that a pie is a divided bar in polar coordinates, we can construct other polar graphics that are less well known. We will also come to realize why a histogram is not a bar chart and why many other graphics that look similar nevertheless have different grammars.... Elegant design requires us to think about a theory of graphics, not charts.”

— Leland Wilkinson, *The Grammar of Graphics, Second.*

but chart typologies *can* help us learn and discuss encodings



resources

References

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Wilkinson, Leland. *The Grammar of Graphics*. Second. Springer, 2005.

supplemental visuals

Doumont applied to data encoding, which works best? — **iterative process of *creating, questioning, testing!***

“

The ceramics teacher announced on opening day that he was dividing the class into two groups. All those on the left side of the studio, he said, would be graded solely on the quantity of work they produced, all those on the right solely on its quality. His procedure was simple: on the final day of class he would bring in his bathroom scales and weigh the work of the “quantity” group: fifty pounds of pots rated an “A”, forty pounds a “B”, and so on. Those being graded on “quality”, however, needed to produce only one pot —albeit a perfect one —to get an “A”.

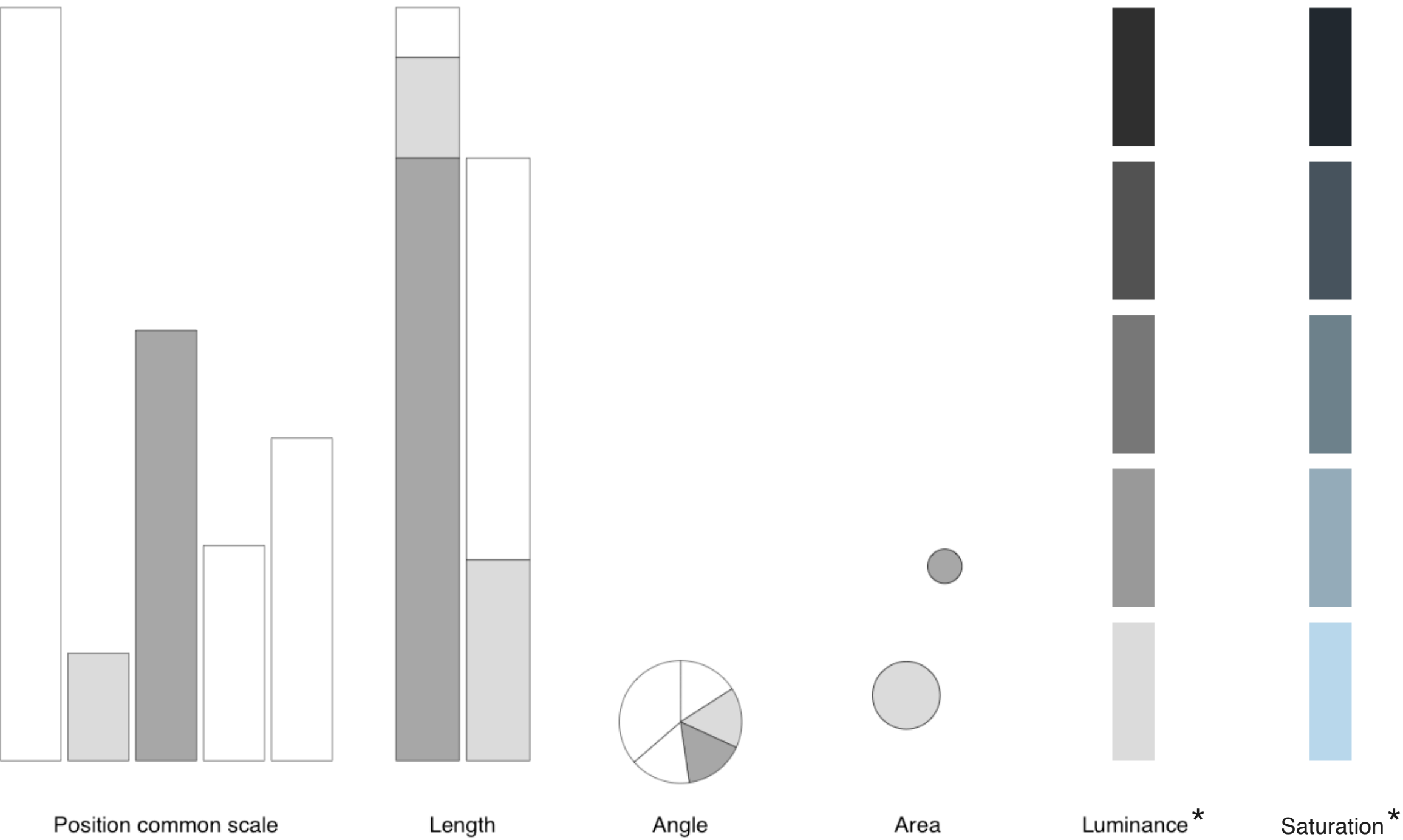
Well, came grading time and a curious fact emerged: **the works of *highest quality* were all produced by the group being graded for *quantity*.** It seems that while the “quantity” group was busily churning out piles of work—and learning from their mistakes —the “quality” group had sat theorizing about perfection, and in the end had little more to show for their efforts than grandiose theories and a pile of dead clay.

”

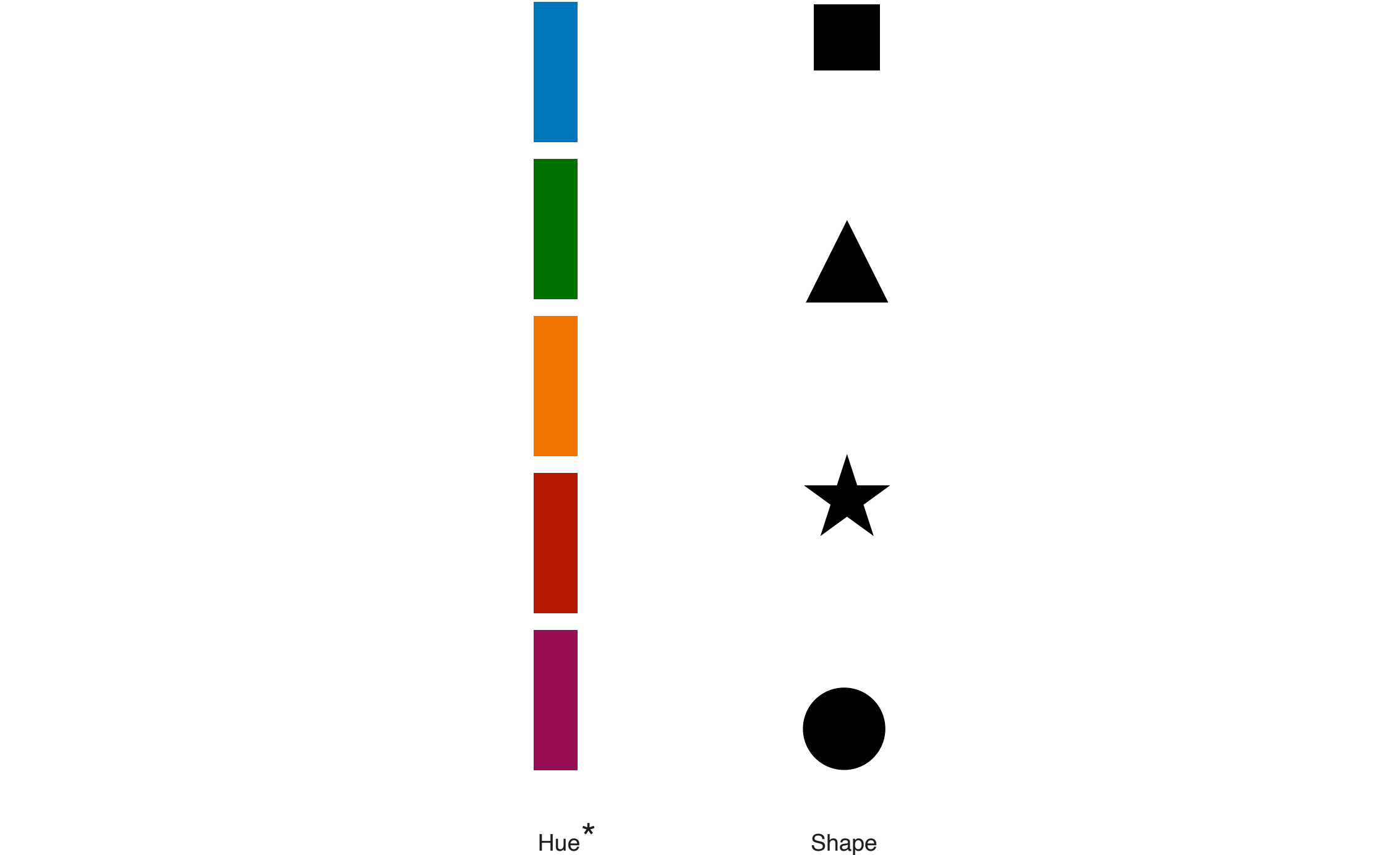
— Bayles and Orland, *Art & Fear. Observations on the Perils (and Rewards) of Artmaking*. The Image Continuum, 1993.

channel effectiveness for encoding data

ratio, interval, and ordered



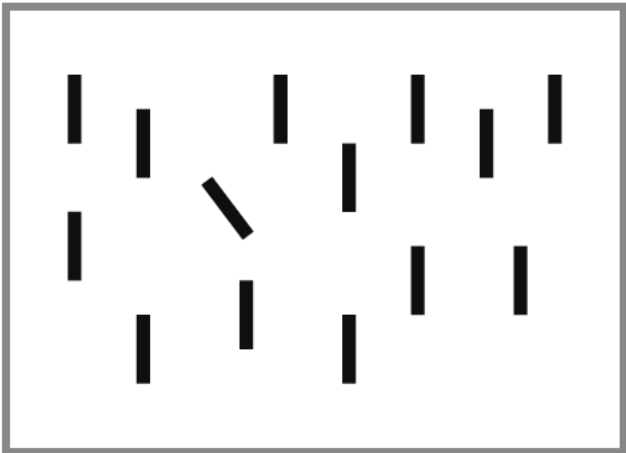
categorical



perceptual psychology

perceptual psychology, *pre-attentive attributes*

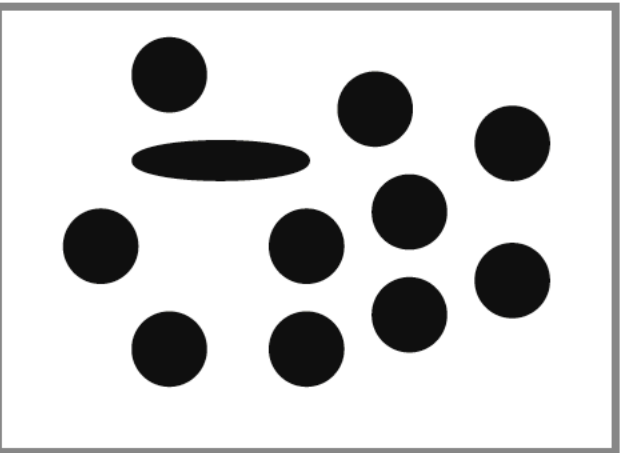
Orientation



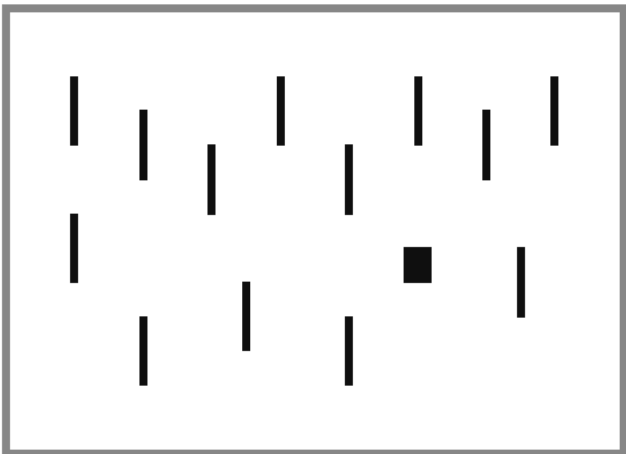
Curved straight



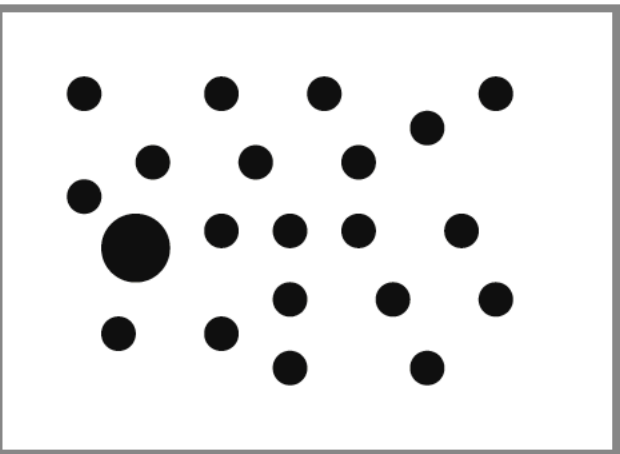
Shape



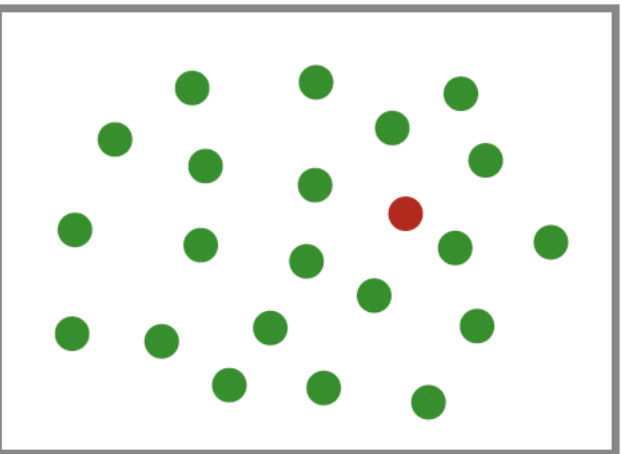
Shape



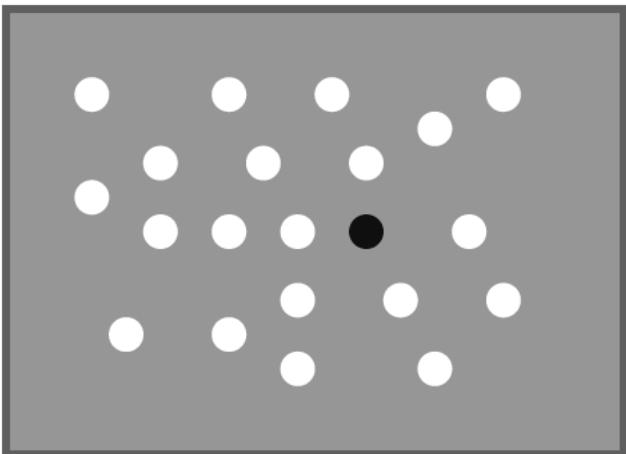
Size



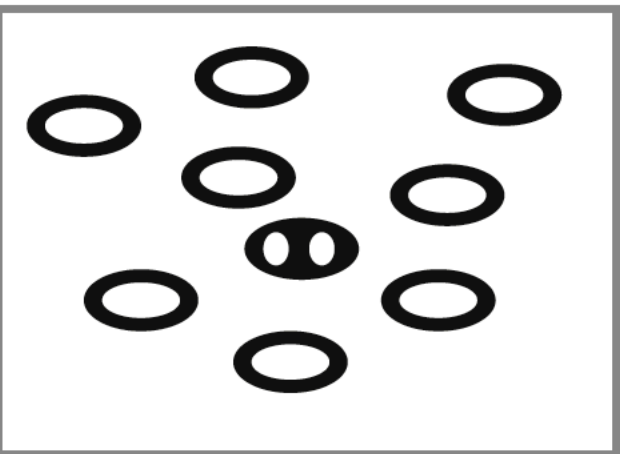
Color



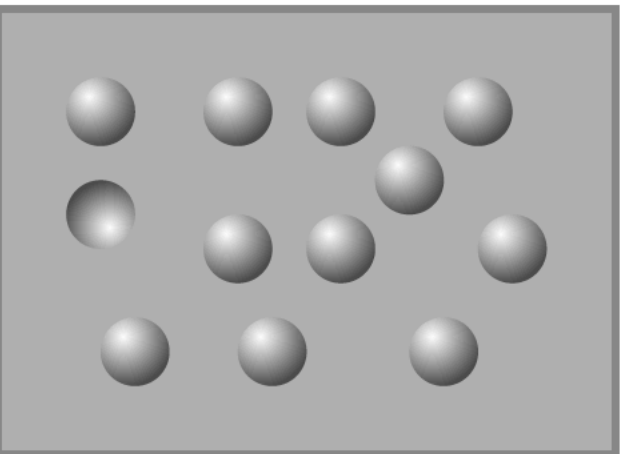
Light/dark



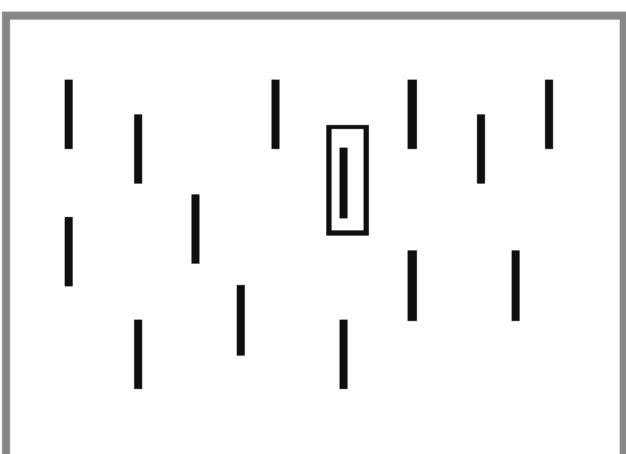
Topology (or count)



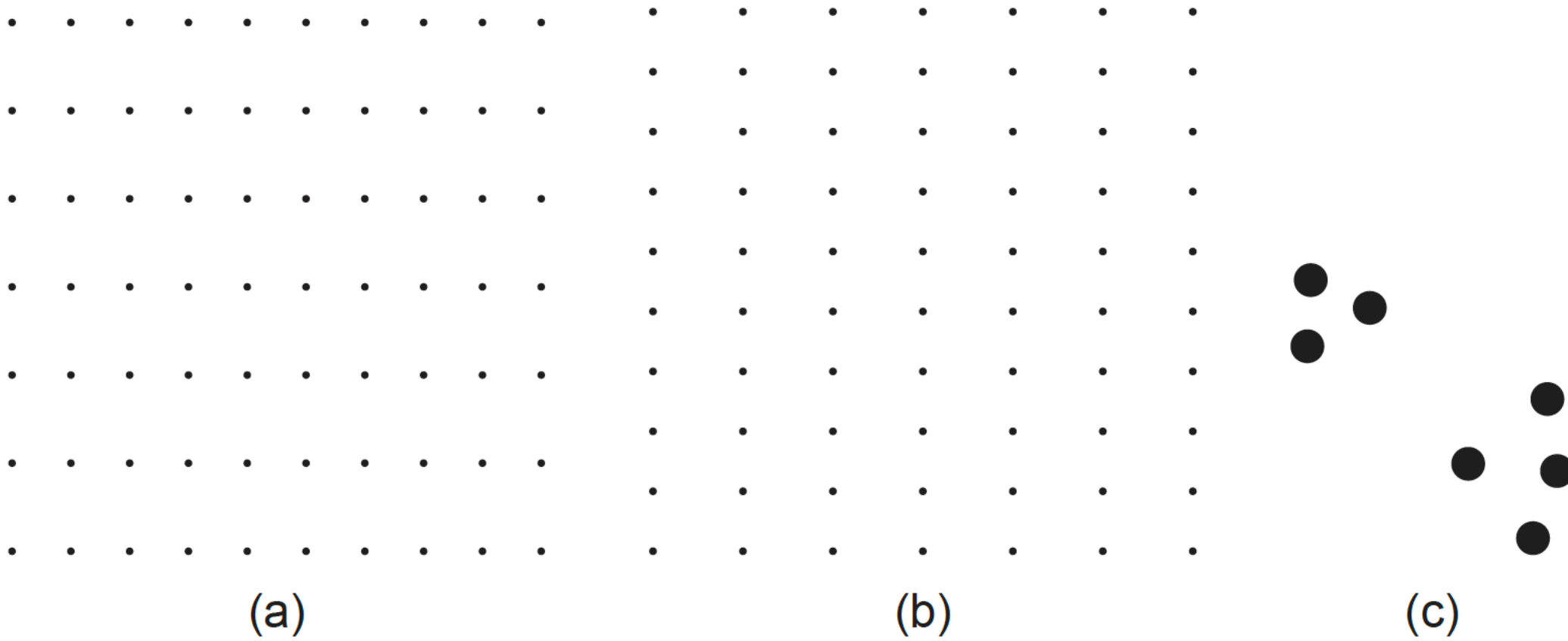
Convex/concave



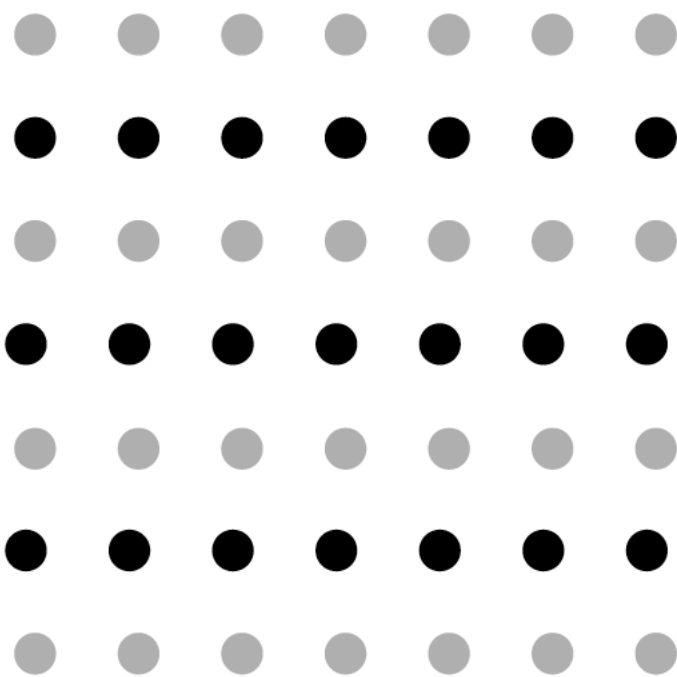
Addition



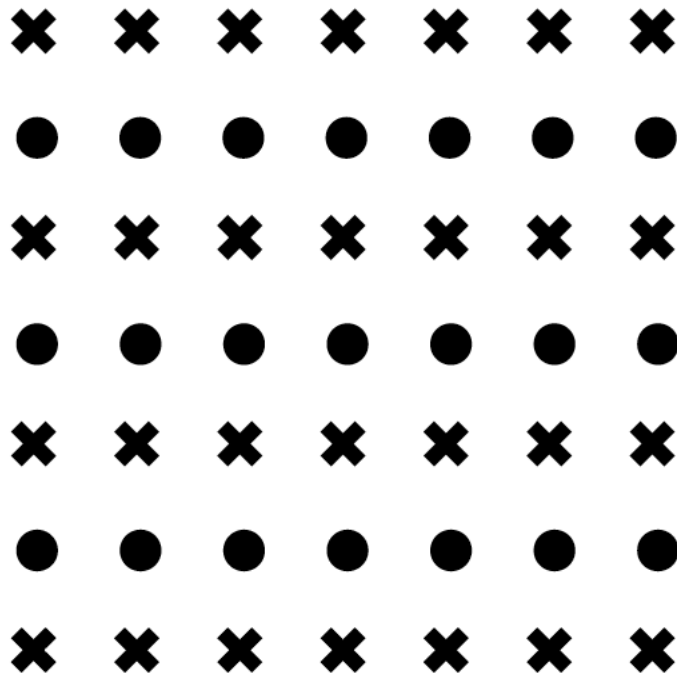
perceptual psychology, Gestalt principles, *proximity*



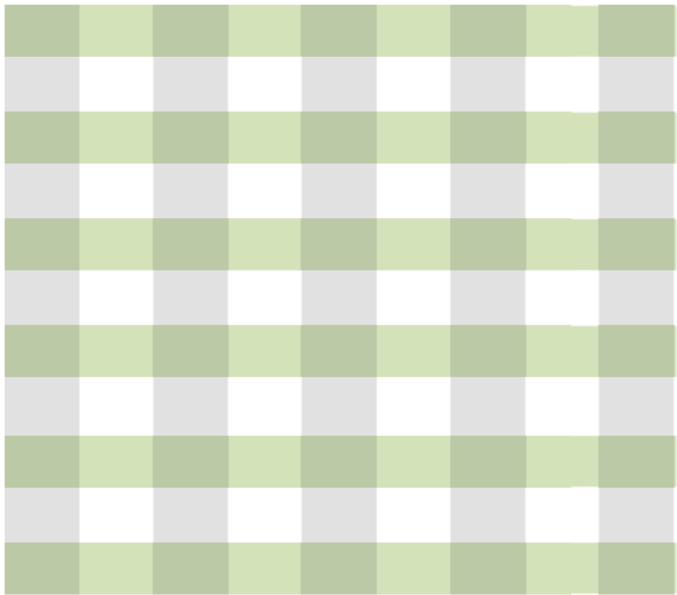
perceptual psychology, Gestalt principles, *similarity*



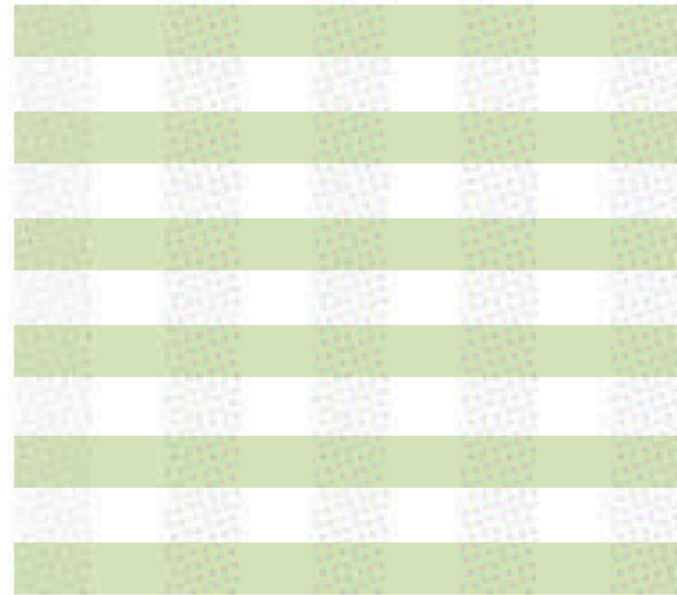
(a)



(b)

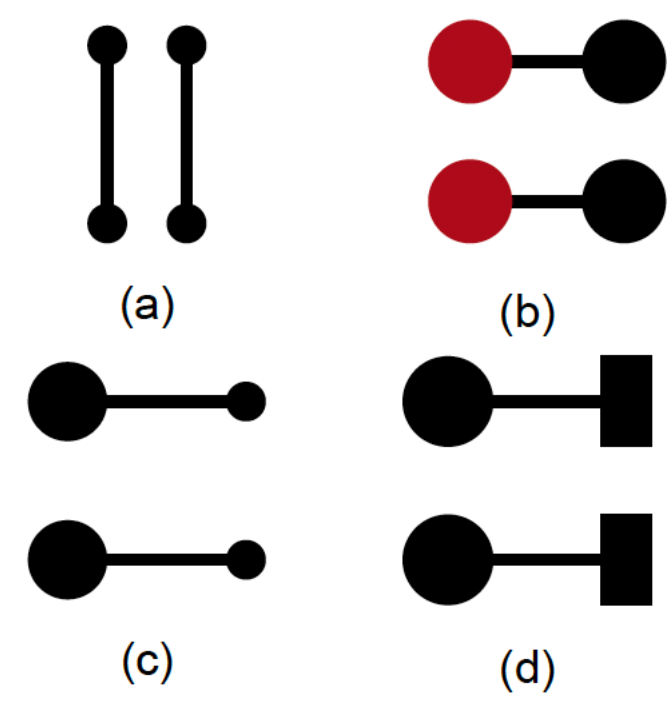


(c)

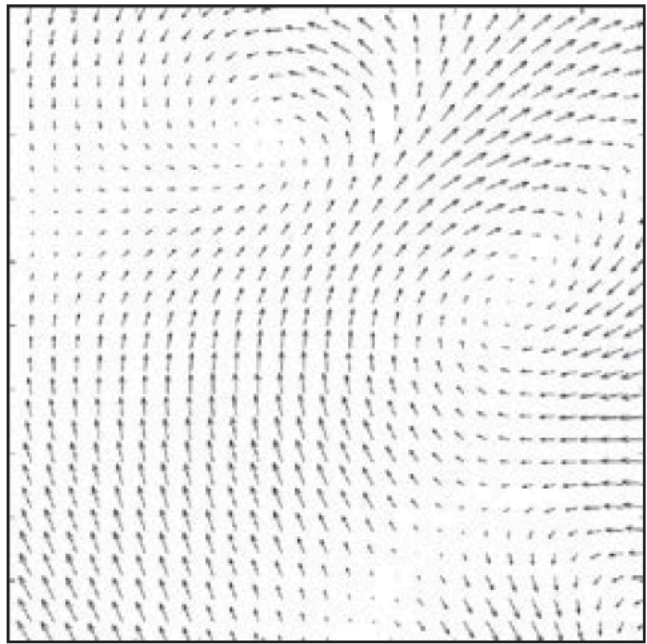


(d)

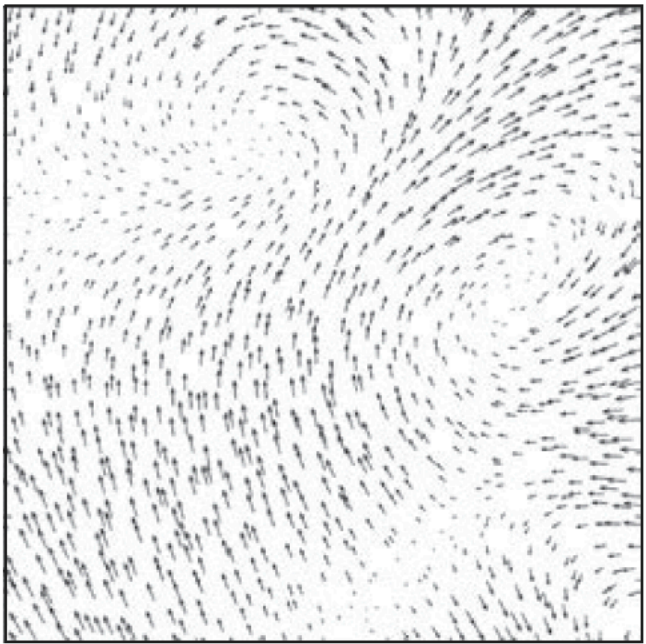
perceptual psychology, Gestalt principles, *connectedness*



perceptual psychology, Gestalt principles, *orientation*, *magnitude*, *direction*



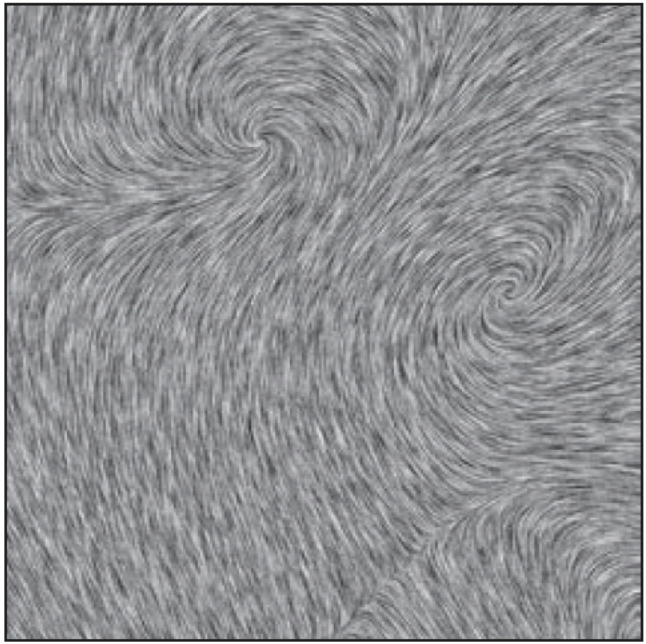
(a)



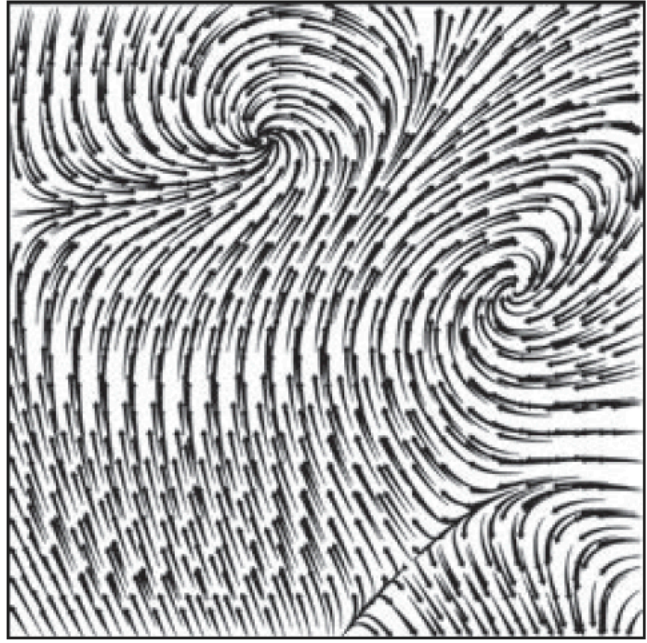
(b)



(c)



(d)



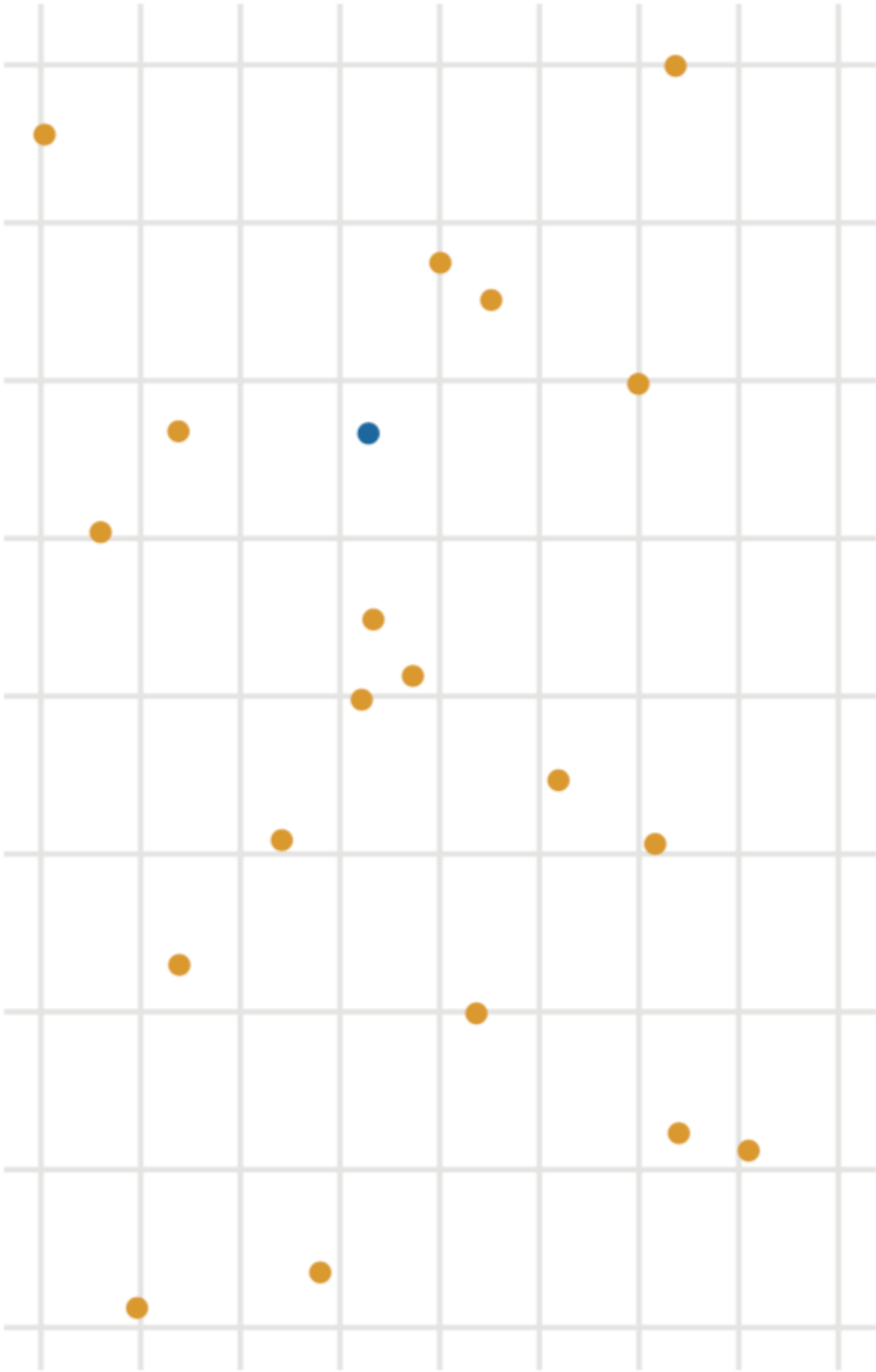
(e)



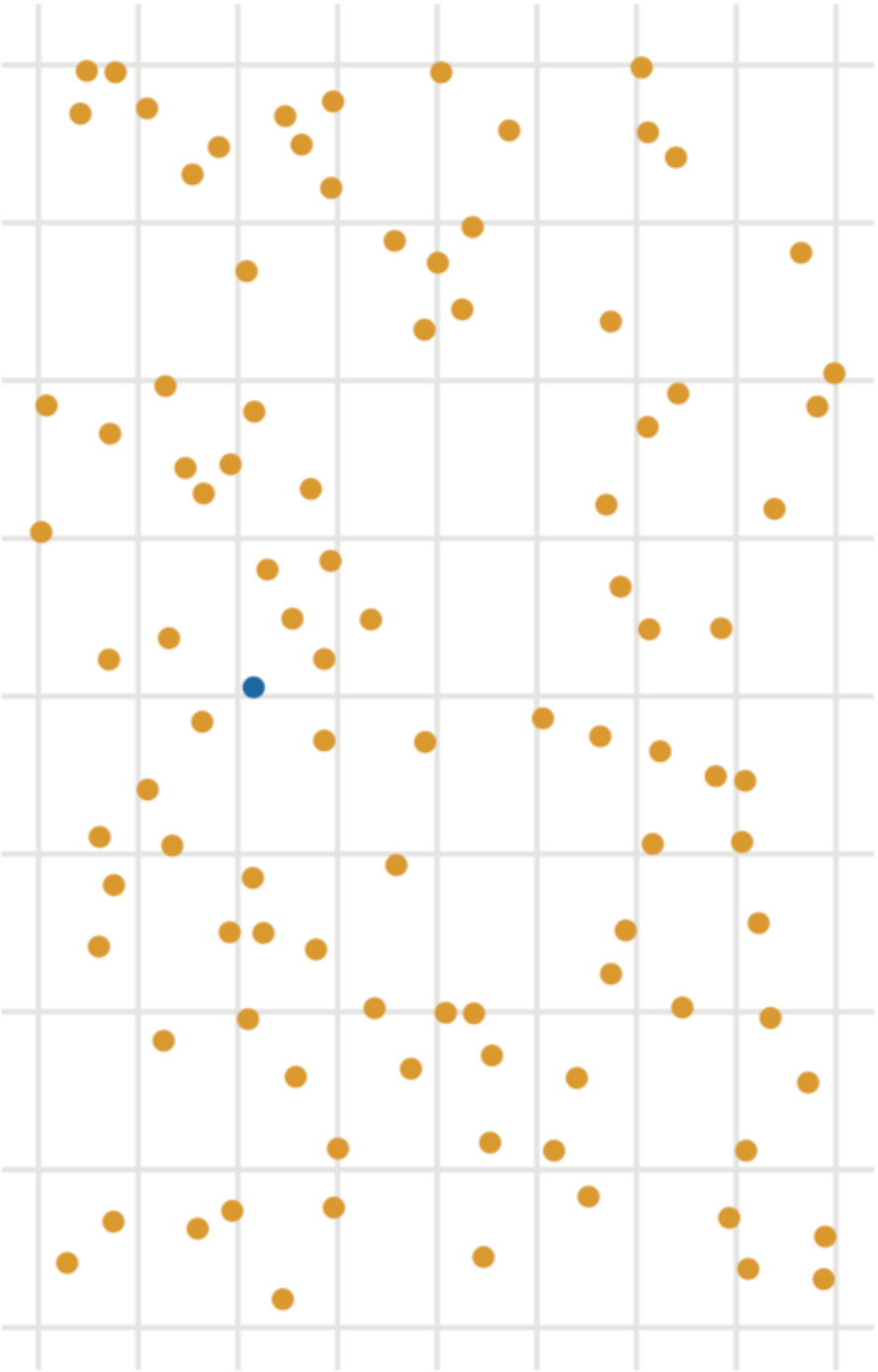
(f)

perceptual psychology, example — *focusing visual attention*

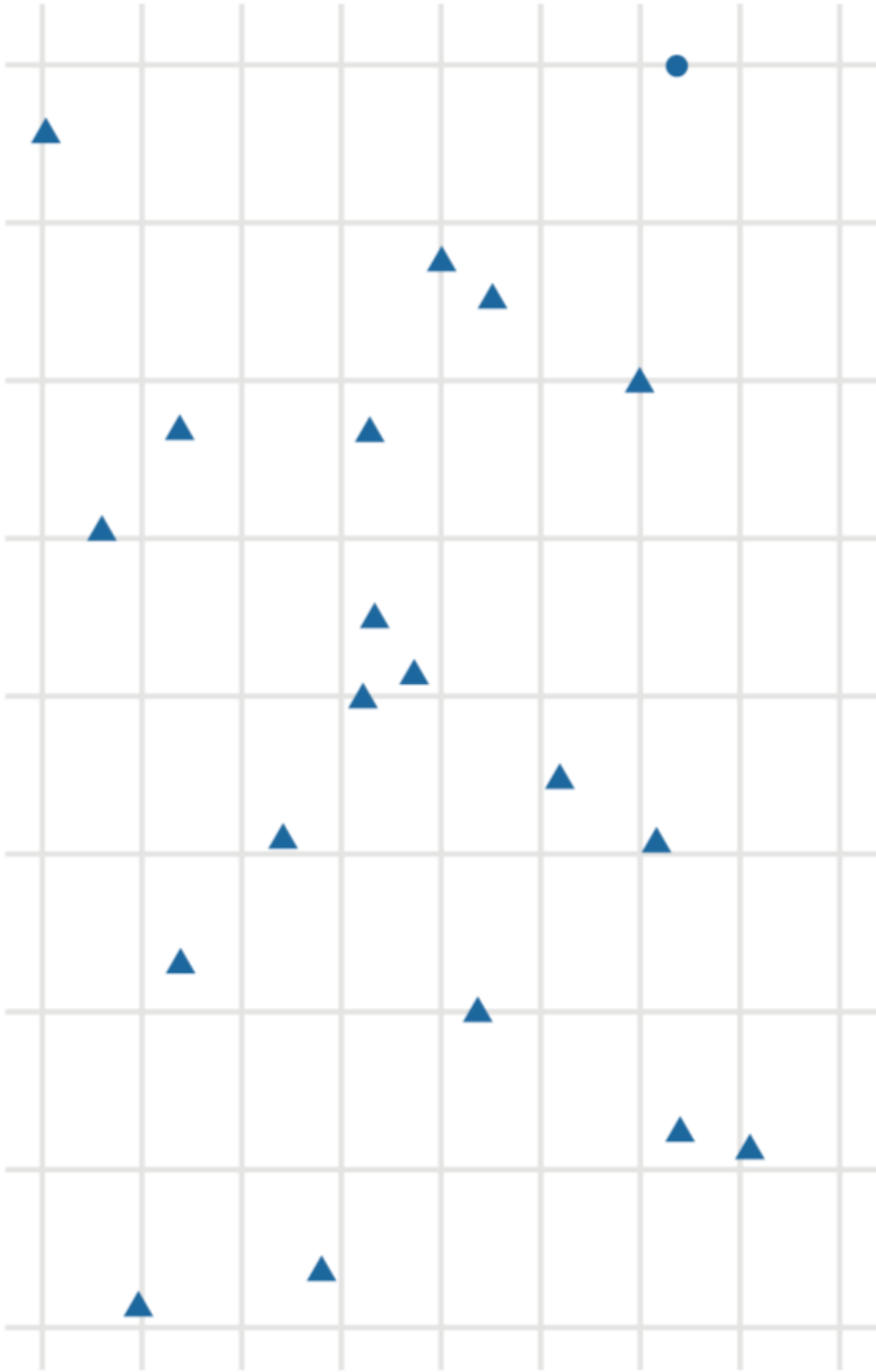
Color only, $N = 20$



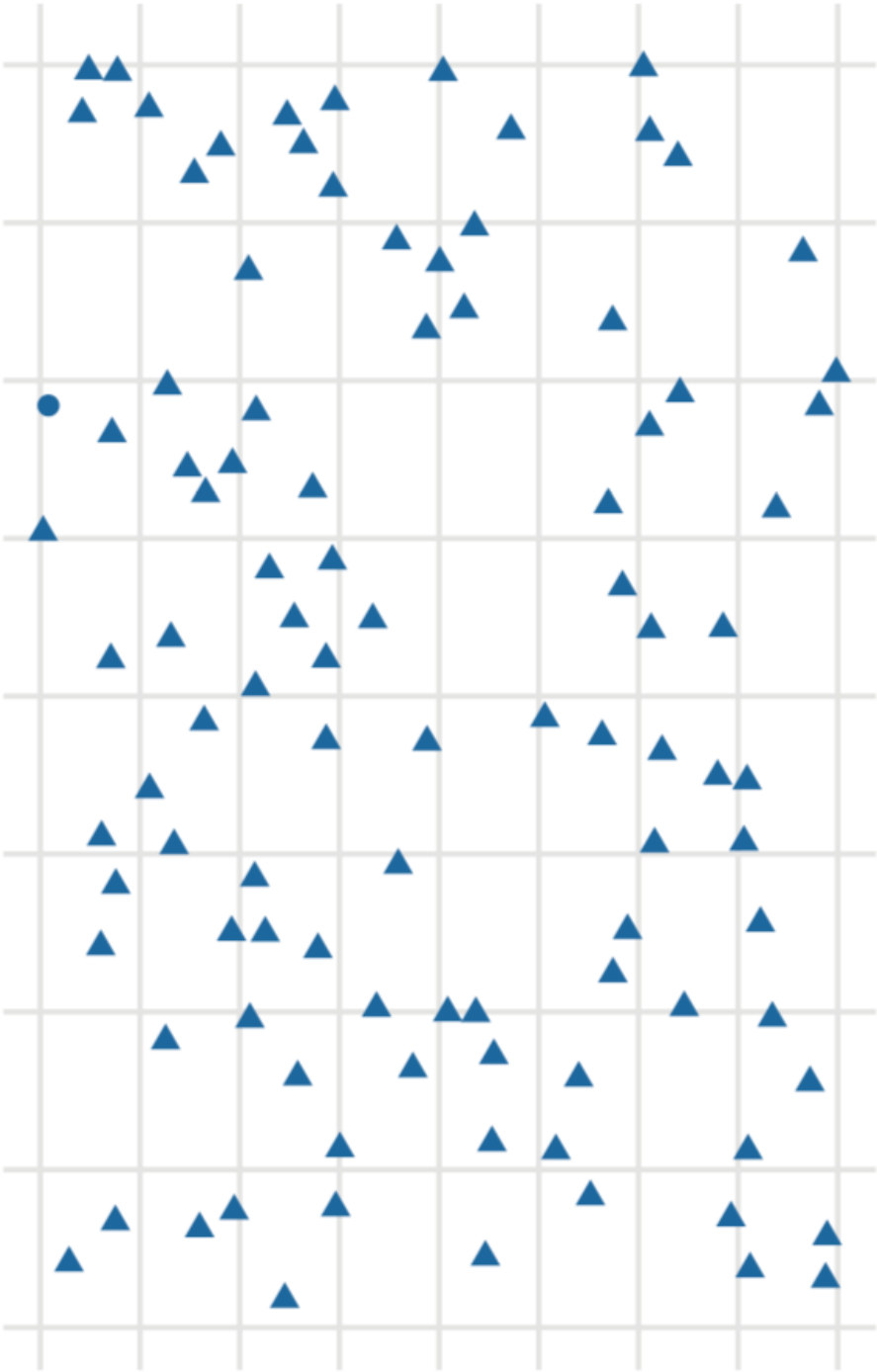
Color only, $N = 100$



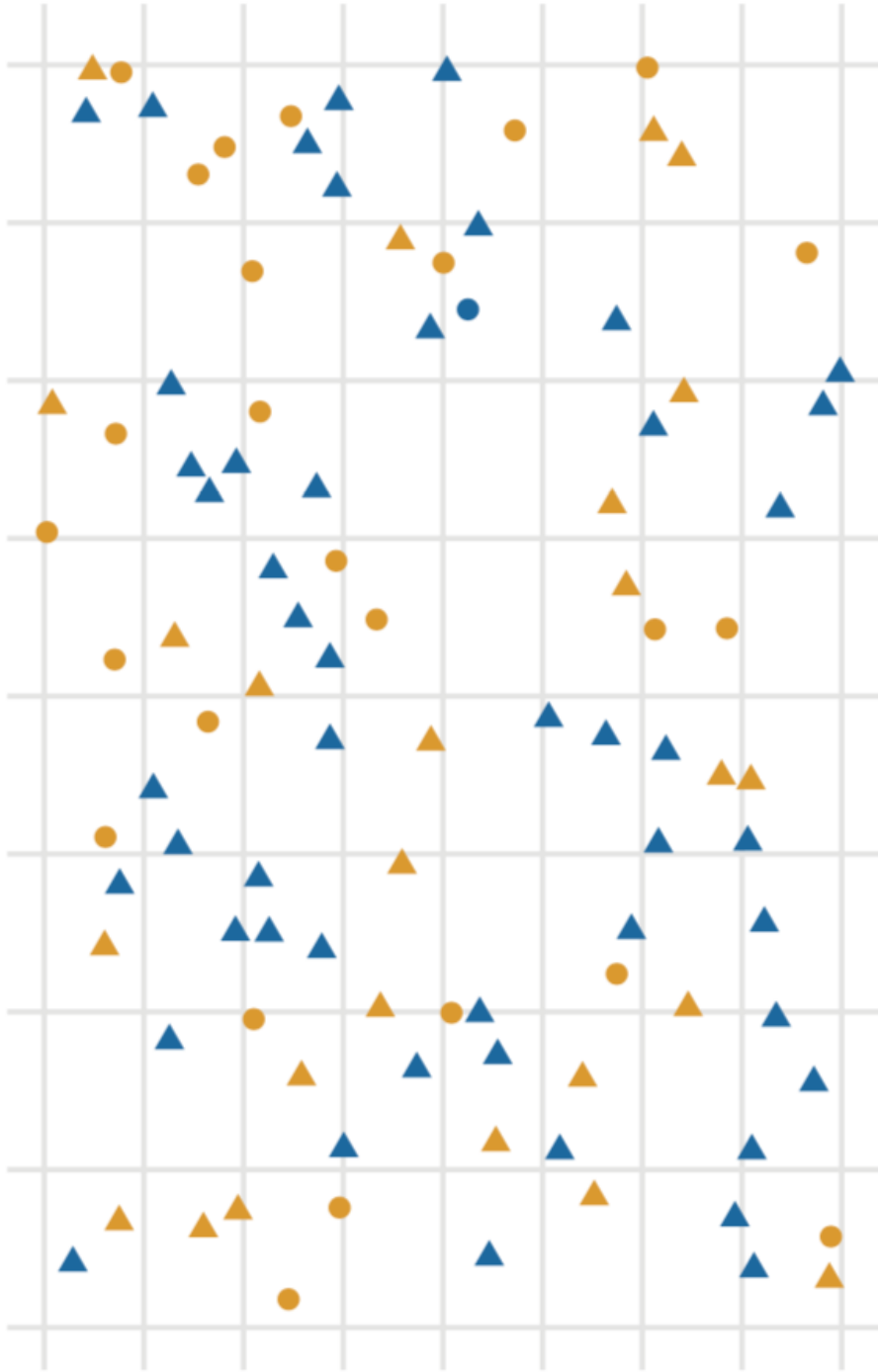
Shape only, $N = 20$



Shape only, $N = 100$

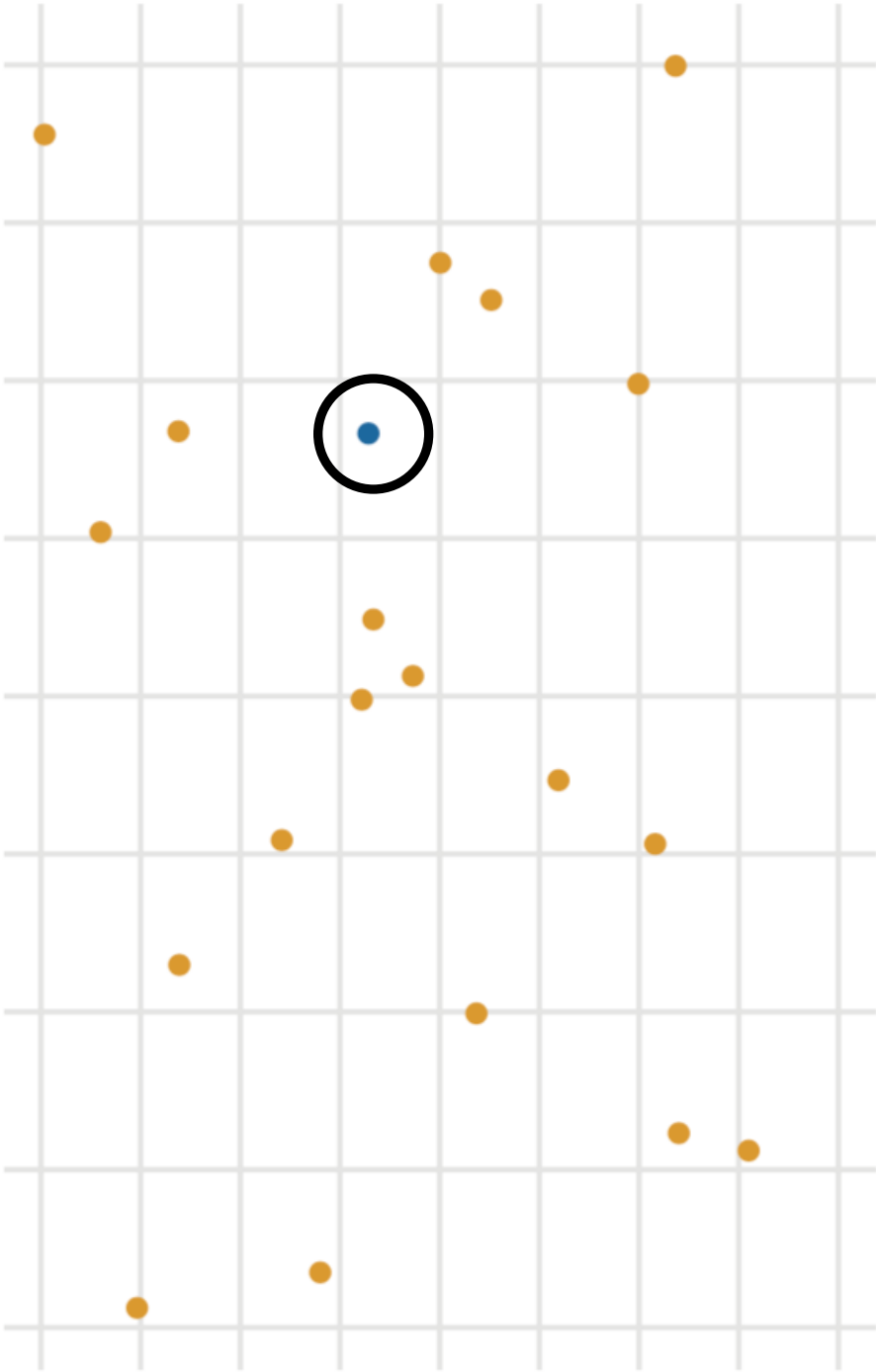


Color & shape, $N = 100$

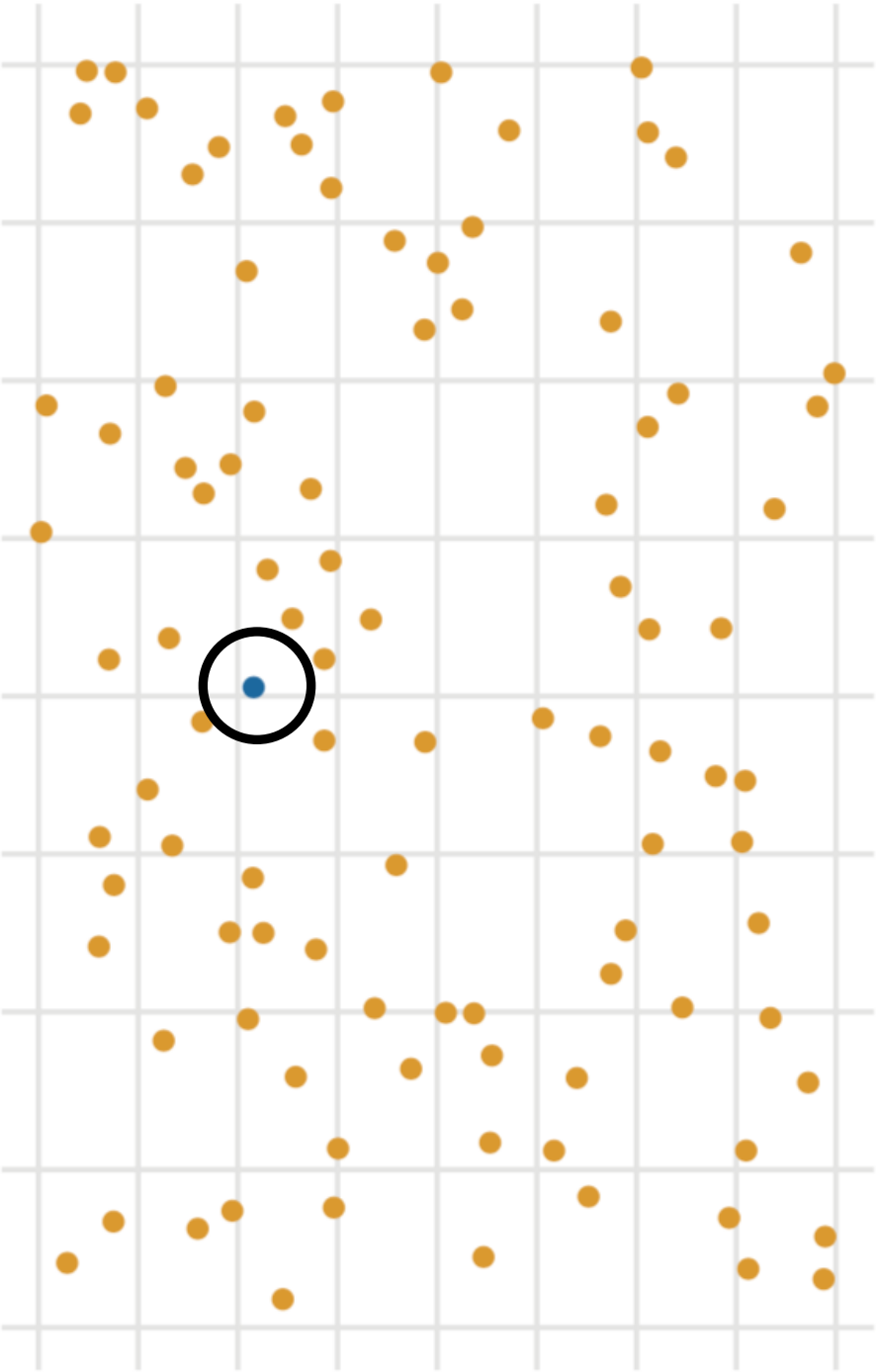


perceptual psychology, example — *focusing visual attention*

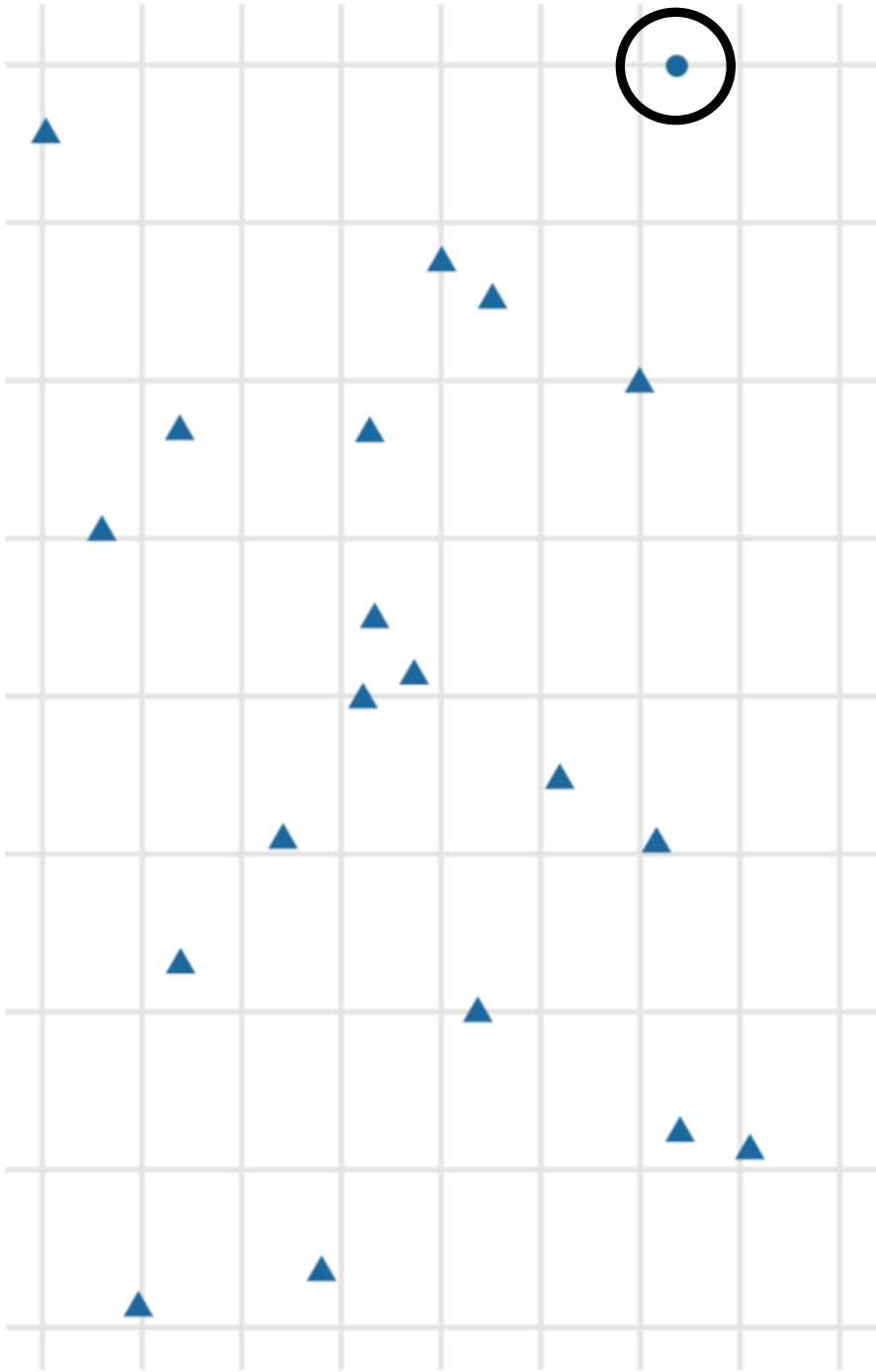
Color only, $N = 20$



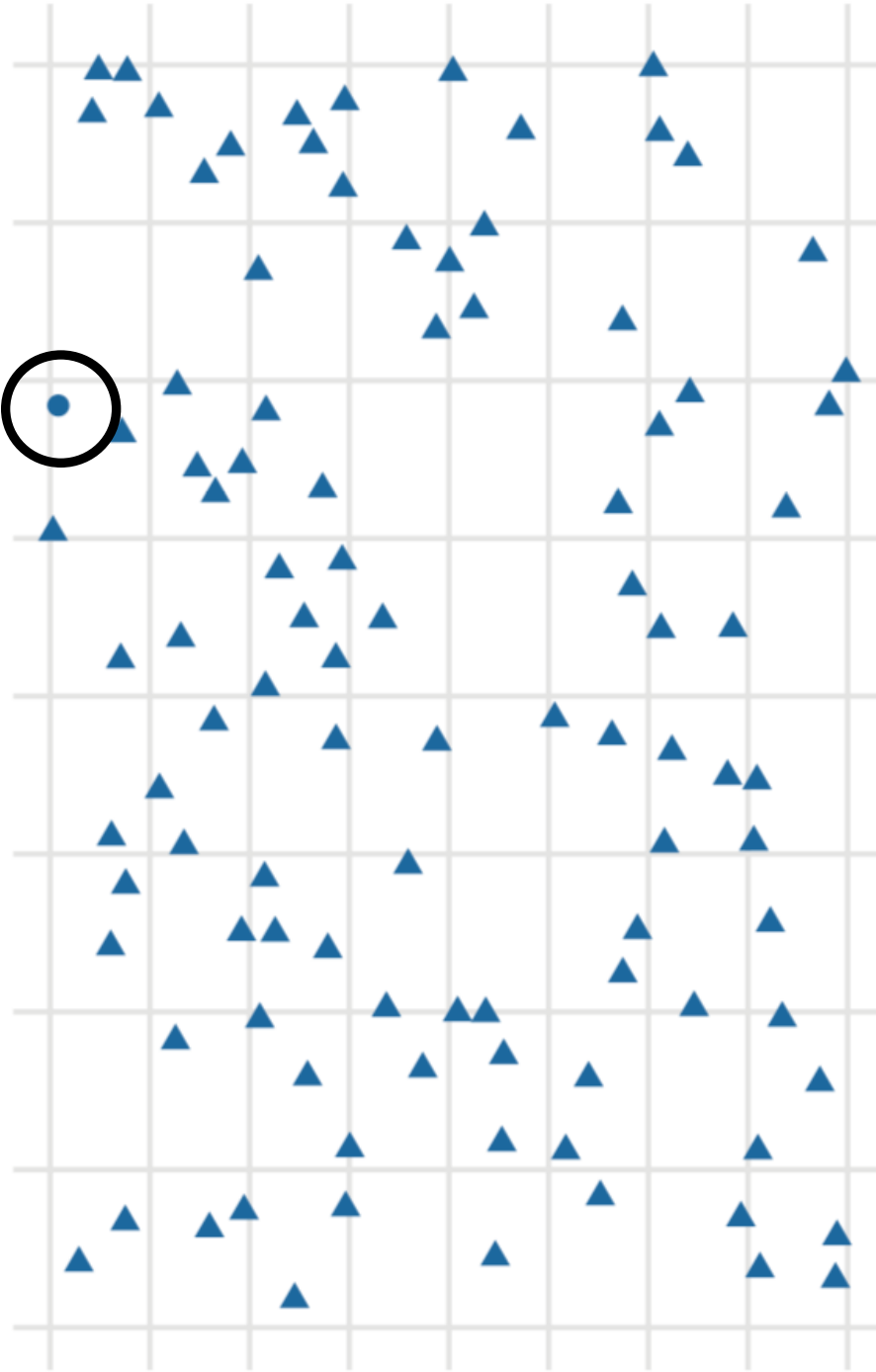
Color only, $N = 100$



Shape only, $N = 20$



Shape only, $N = 100$



Color & shape, $N = 100$

